SECTION 3

DESIGN OF POST-TENSIONED COMPONENTS FOR FLEXURE

DEVELOPED BY THE PTI EDC-130 EDUCATION COMMITTEE
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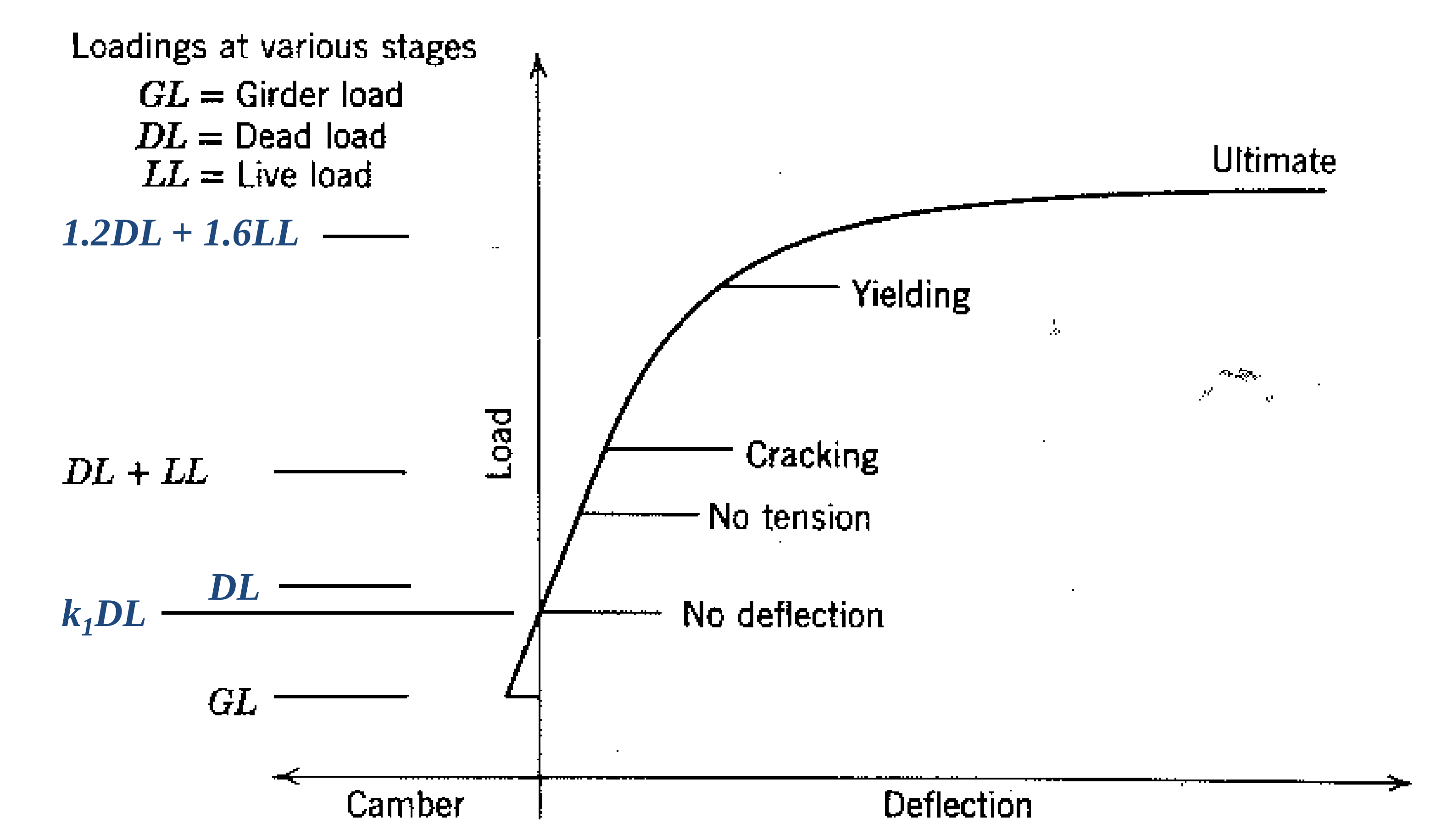
NOTE: MOMENT DIAGRAM CONVENTION

- In PT design, it is preferable to draw moment diagrams to the tensile face of the concrete section. The tensile face indicates what portion of the beam requires reinforcing for strength.
- When moment is drawn on the tension side, the diagram matches the general drape of the tendons. The tendons change their vertical location in the beam to follow the tensile moment diagram. Strands are at the top of the beam over the support and near the bottom at mid span.
- For convenience, the following slides contain moment diagrams drawn on both the tensile and compressive face, denoted by (T) and (C), in the lower left hand corner. Please delete the slides to suit the presenter's convention.

OBJECTIVE

- 1 hour presentation
- Flexure design considerations

PRESTRESSED GIRDER BEHAVIOR

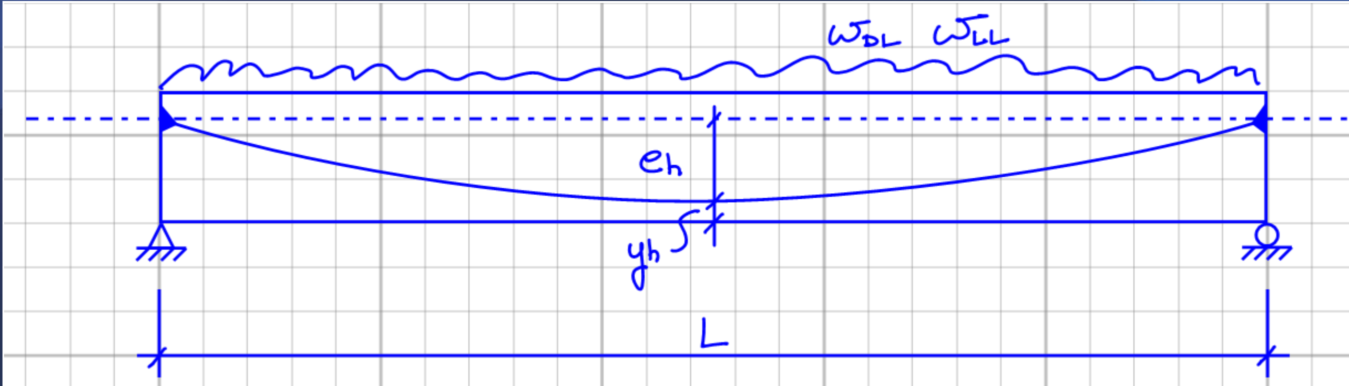


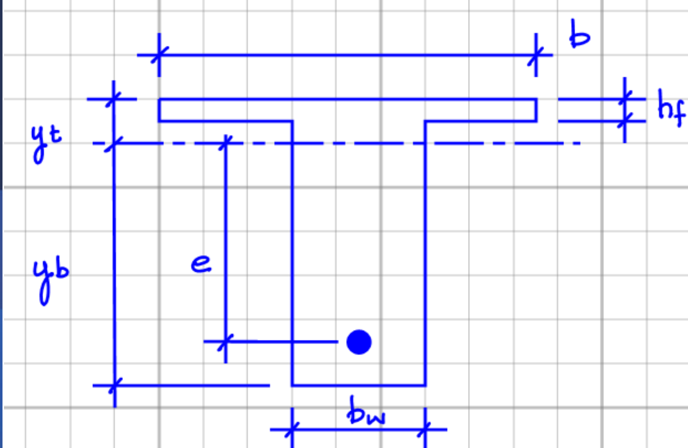
Lin and Burns, Design of Prestressed Concrete Structures, 3rd Ed., 1981

LIMIT STATES – AND PRESENTATION OUTLINE

- Load Balancing – k_1DL
  - Minimal deflection.
    Select k1 to balance majority of sustained load.
- Service – *DL + LL*
  - Concrete cracking.
    Check tension and compressive stresses.
- Strength – *1.2DL + 1.6LL +1.0Secondary*
  - Ultimate strength.
    Check Design Flexural Strength (ϕM_n)

EXAMPLE





7.4 DESIGN OF A SINGLE SPAN CAST-IN-PLACE T-BEAM

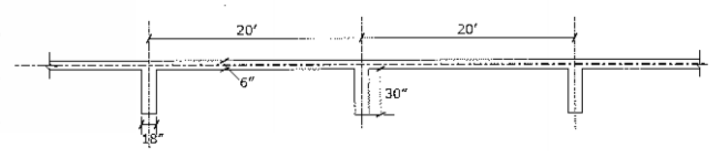


Fig. 7.10 Cross-Section of the Beam/Slab System

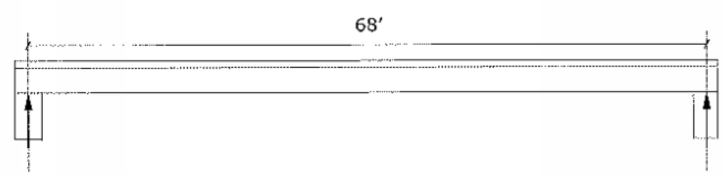


Fig. 7.11 Beam Elevation

DIMENSIONS AND PROPERTIES

Geometry and section properties

$L = 68ft$	end to end of girder and assumed center-to-center span length
$bw = 18in$	web width
$h = 36in$	section height
$hf = 6in$	flange thickness
$s = 20ft$	beam spacing

Tendon data

$A_{ps} = 28 \cdot Aps5$	Number of 0.5 in. dia. strands in fully bonded tendon
$y_h = 3.75in$	Distance from bottom of beam to cgs of tendon at low point of harp.

Material properties

$f'_c = 5000psi$	Specified 28-day concrete compressive strength.
$f'_{ci} = 4000psi$	Specified concrete compressive strength at prestress transfer.
$f_y = 60ksi$	Specified yield strength of mild steel reinforcement
$f_{pu} = 270ksi$	Specified ultimate tensile strength of prestressing strand

SECTION PROPERTIES

Determine effective flange width according to ACI 8.12.2.

$$16 \cdot hf + bw = 114 \cdot in$$

$$(s - bw) \cdot 0.5 = 111 \cdot in$$

$$0.25 \cdot L = 204 \cdot in$$

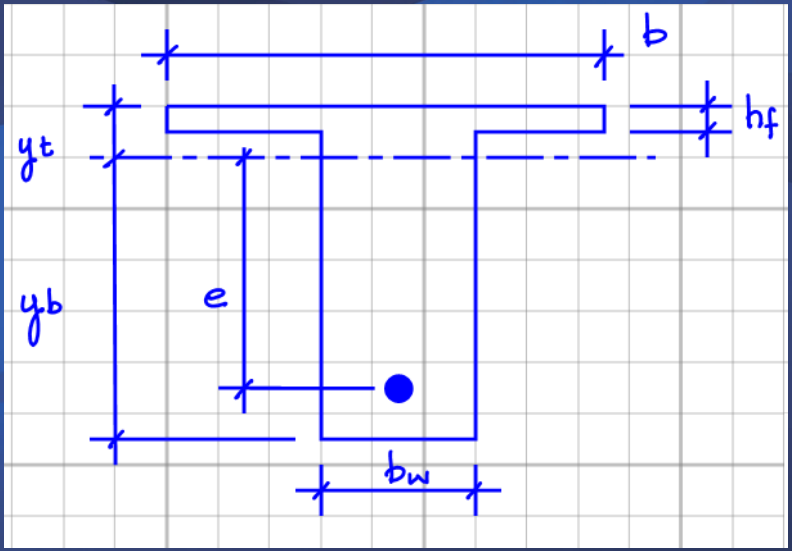
Effective flange width is 111 in.

$$I = 139118 \cdot in^4$$

$$A = 1206 \cdot in^2$$

$$y_b = 24.94 \cdot in$$

$$y_t = 11.06 \cdot in$$



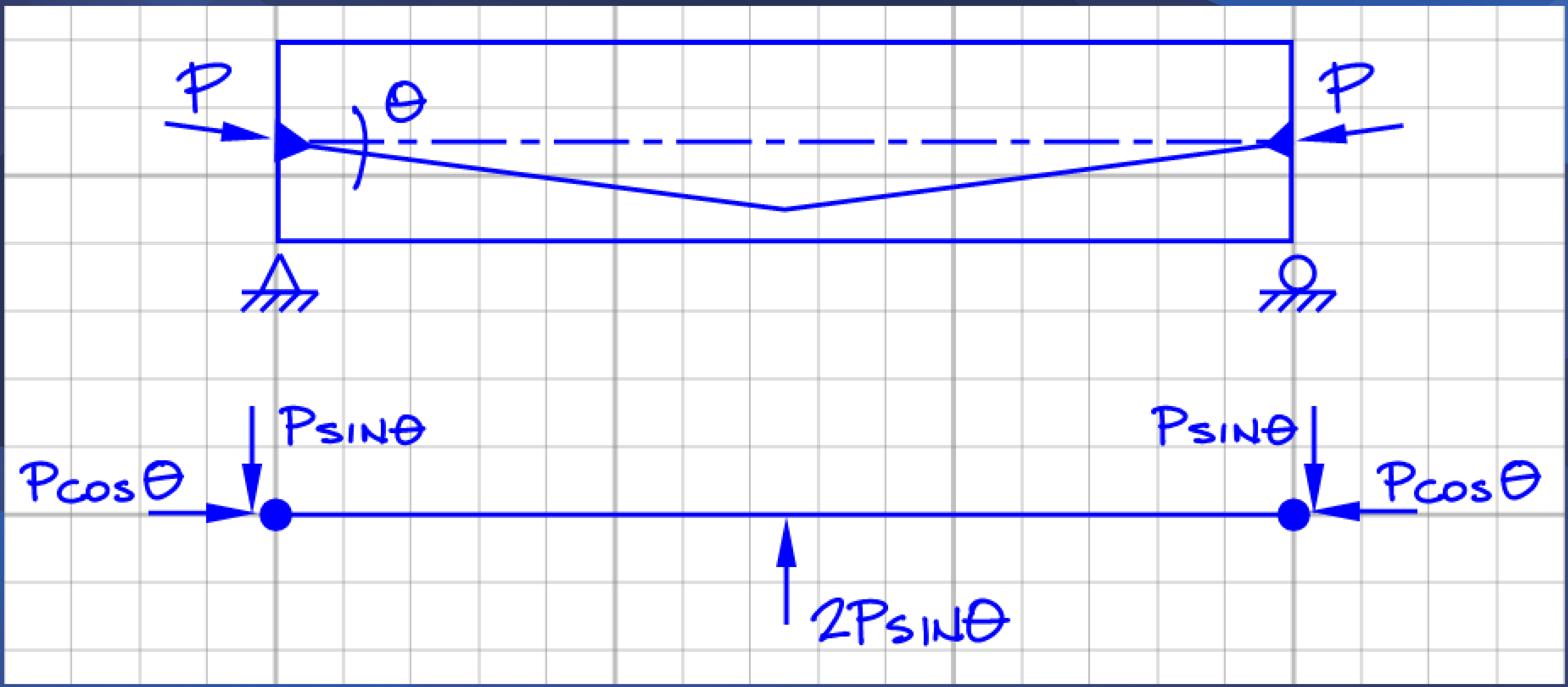
LOAD BALANCING

- Tendons apply external self-equilibrating transverse loads to member.
- Forces applied through anchorages
- The angular change in tendon profile causes a transverse force on the member

LOAD BALANCING

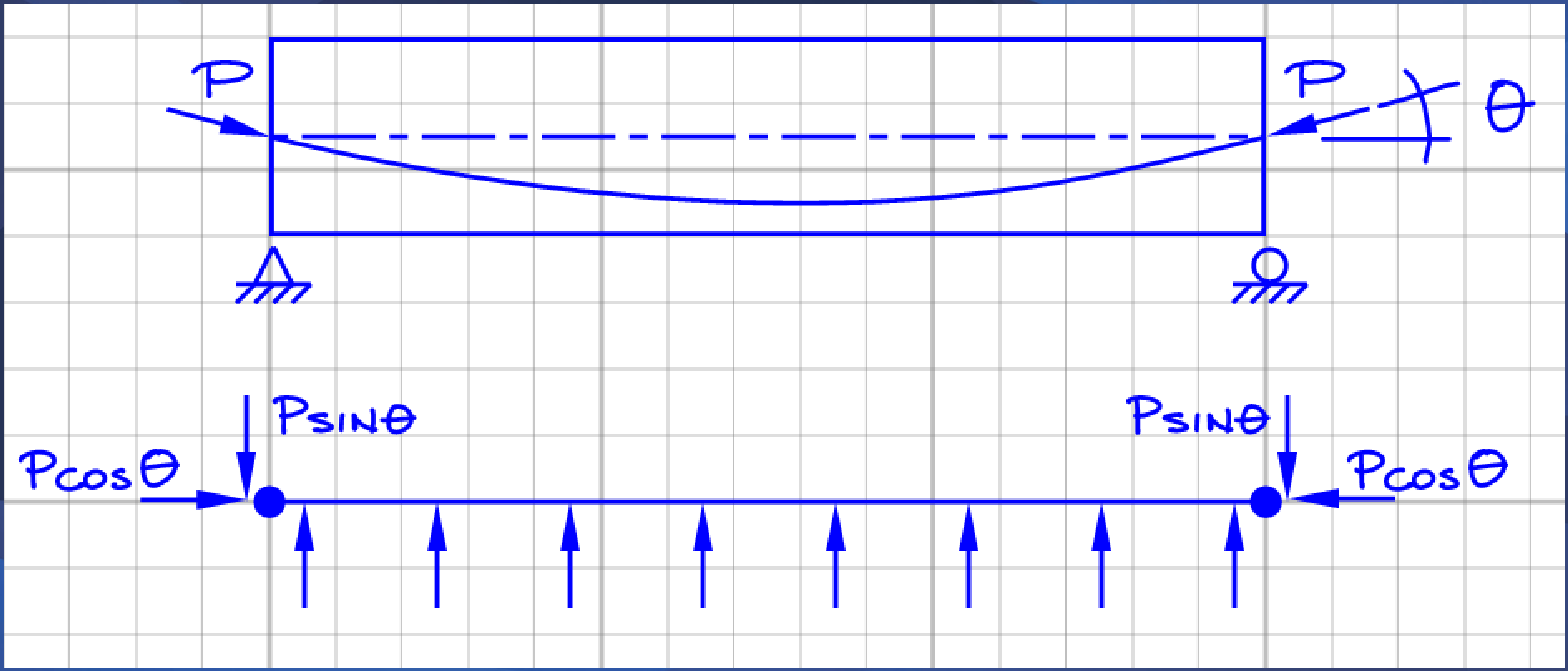
- Transverse forces from tendon “balances” structural dead loads.
- Moments caused by the equivalent loads are equal to internal moments caused by prestressing force

EQUIVALENT LOAD



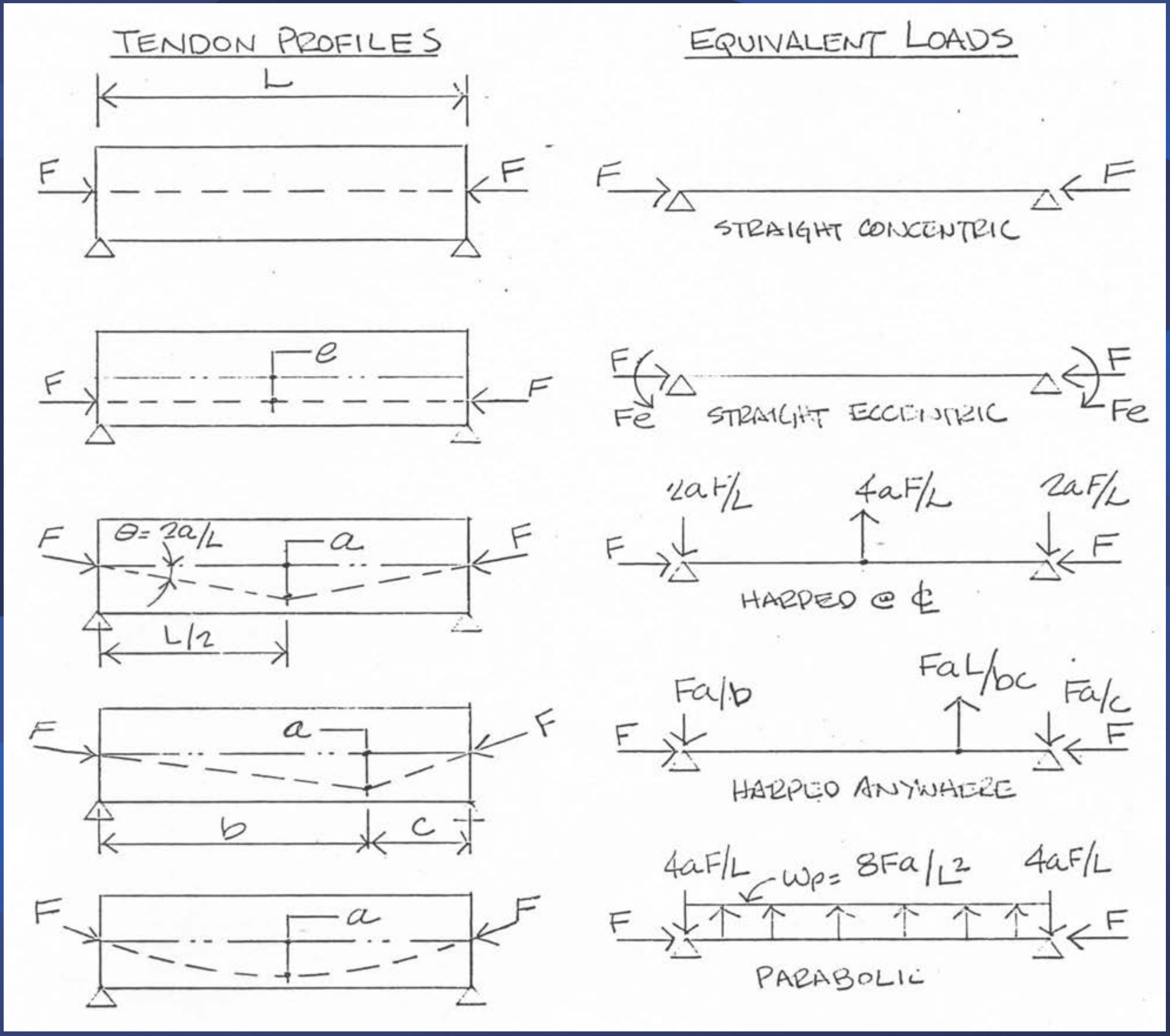
Harped Tendon can be sized and placed such that the upward force exerted by the tendon at midspan exactly ***balances*** the applied concentrated load

EQUIVALENT LOAD



Parabolic Tendon can be sized and placed such that the upward force exerted by the tendon along the length of the member exactly ***balances*** the applied uniformly distributed load

FORMULAS (FROM PTI DESIGN MANUAL?)



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EXAMPLE – LOAD BALANCING

- Determine portion of total dead load balanced by prestressing
- w_{DL} = 0.20 klf
  superimposed dead load (10psf*20ft)
- w_{sw} = 2.06 klf
  self weight including tributary width of slab

EXAMPLE – LOAD BALANCING

$f_{se} = 175\,\text{ksi}$ effective prestress Including all short and long term losses

$A_{ps} = 4.28\,\text{in}^2$ area of 28 0.5 in. dia. strand

$e_c = 21.19\,\text{in}$ eccentriciaty of tendon from section centroid

$$w_{eq} = \frac{8 \cdot (f_{se} \cdot A_{ps}) \cdot e_c}{L^2}$$

$$w_{eq} = 2.29 \cdot \text{klf}$$

$$\frac{w_{eq}}{wDL + w_{sw}} = 101\,\%$$

prestressing balances full dead load

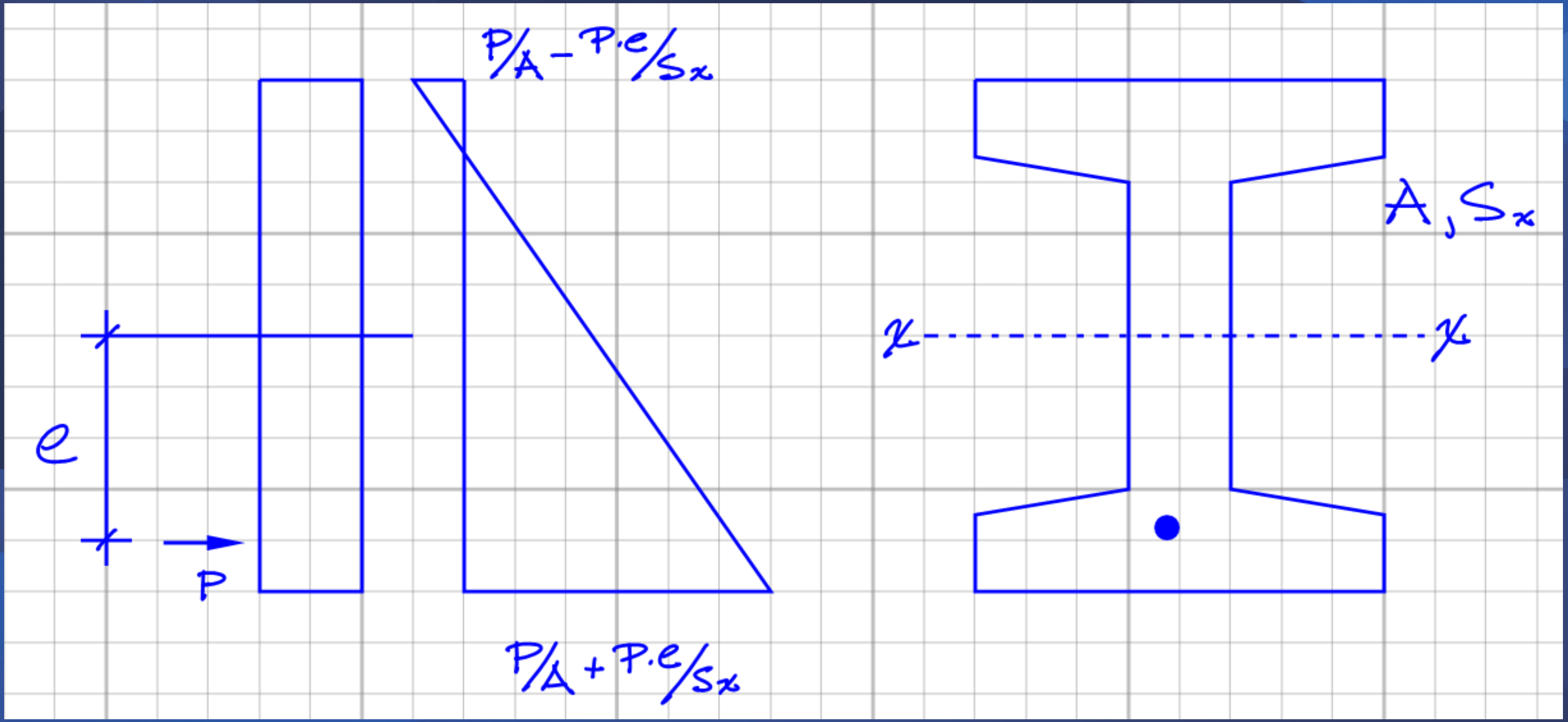
EXAMPLE – LOAD BALANCING

- Prestressing in this example balances ~100% of total dead load.
- In general, balance 65 to 100% of the self-weight
- Balancing in this range ***does not guarantee*** that service or strength limit states will be met. These must be checked separately

STRESSES

- Section remains uncracked
- Stress-strain relationship is linear for both concrete and steel
- Use superposition to sum stress effect of each load. Prestressing is just another load.

CALCULATING STRESSES



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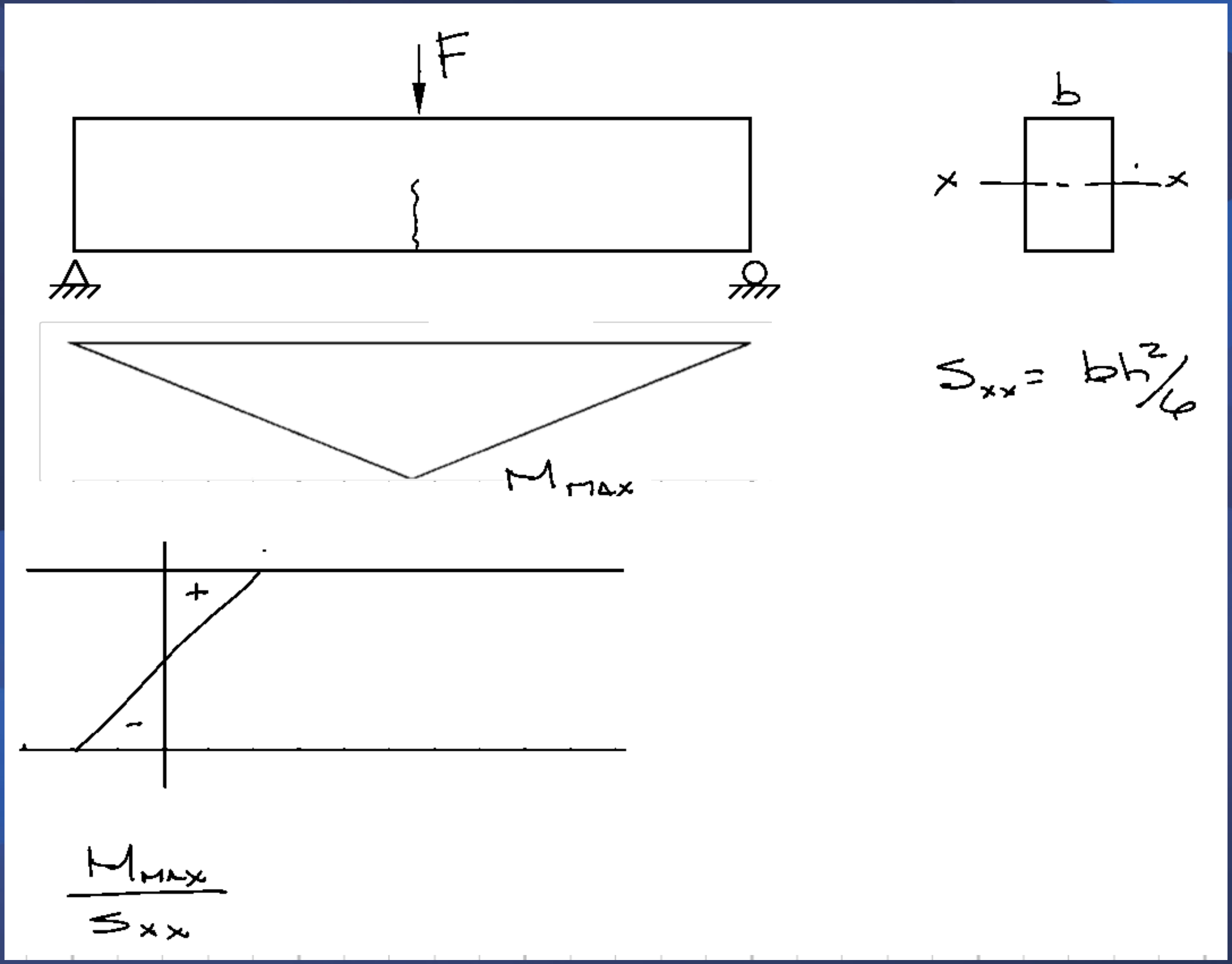
STRESSES

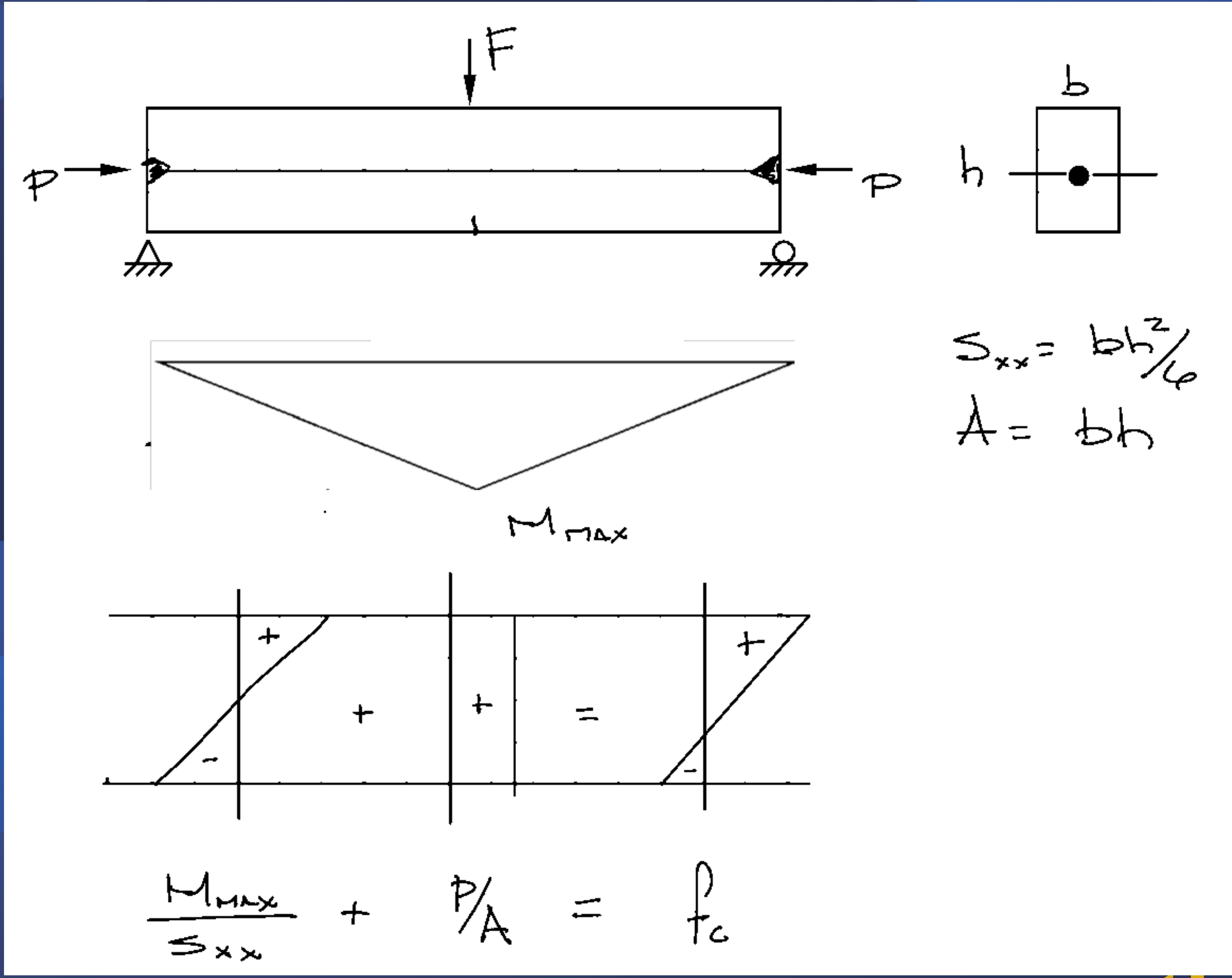
- Stresses are typically checked at significant stages
- The number of stages varies with the complexity and type of prestressing.
- Stresses are usually calculated for the service level loads imposed (i.e. load factors are equal to 1.0). This includes the forces imposed by the prestressing.

PLAIN CONCRETE

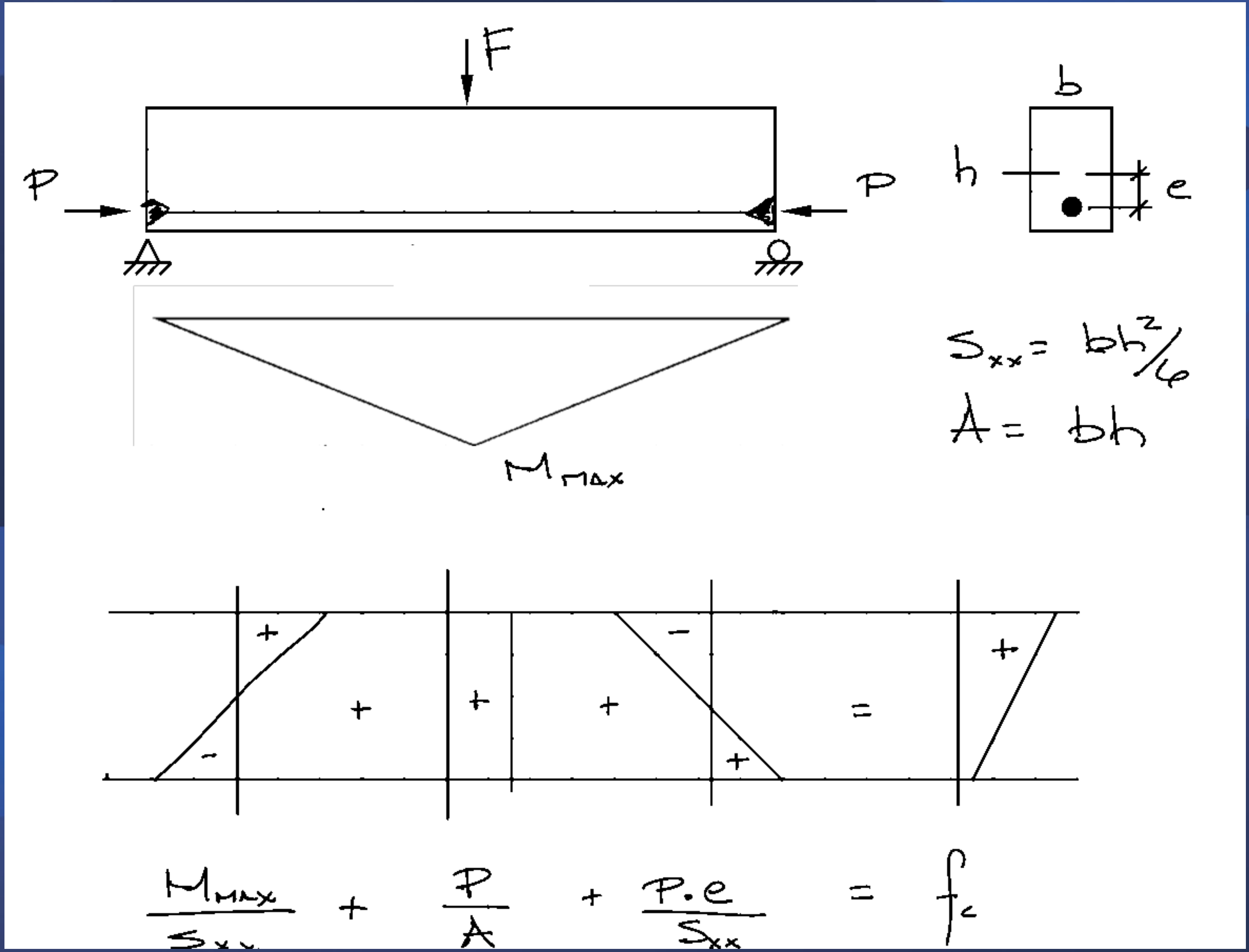


(T)

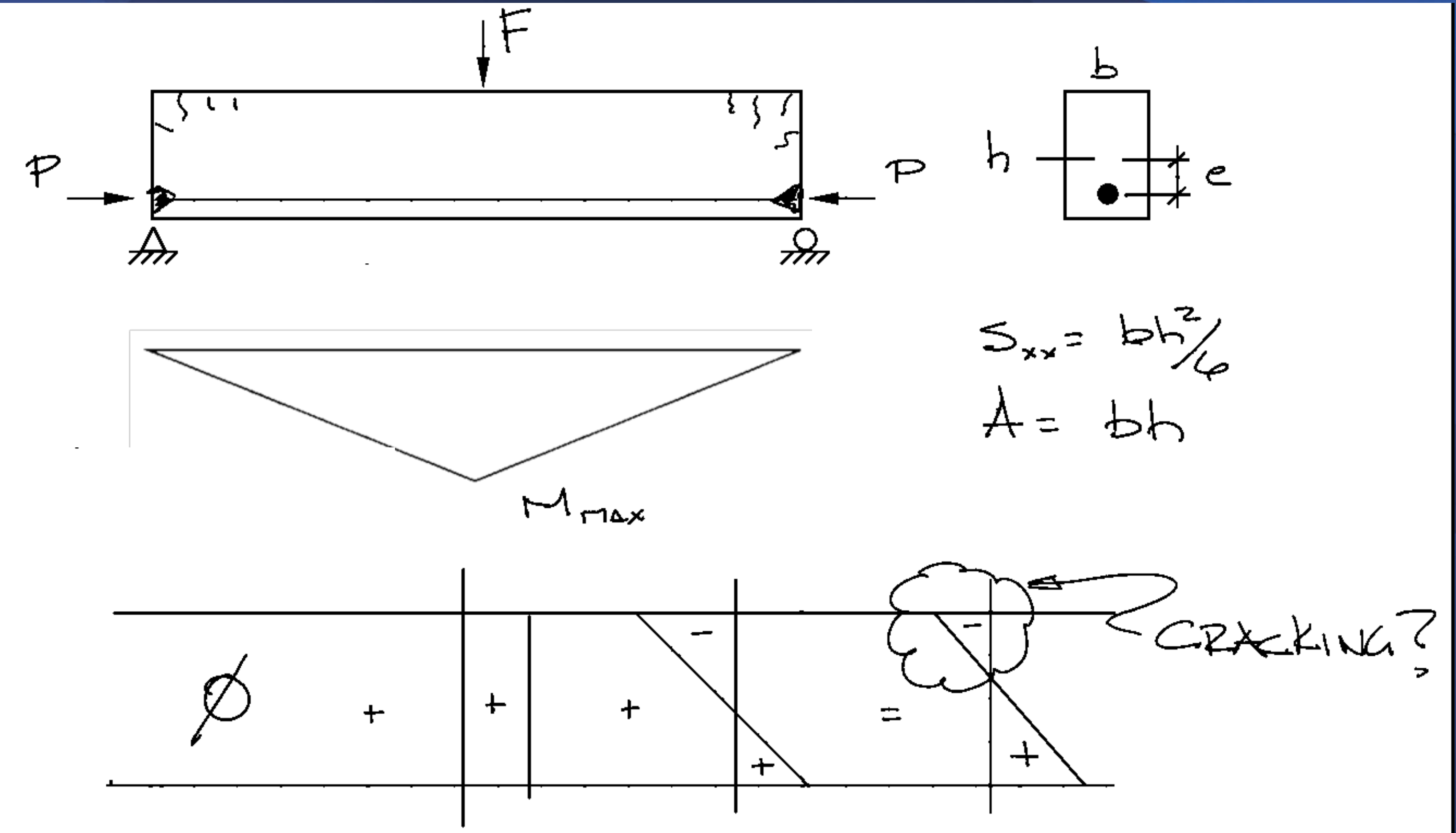
CONCENTRIC PRESTRESSING



ECCENTRIC PRESTRESSING

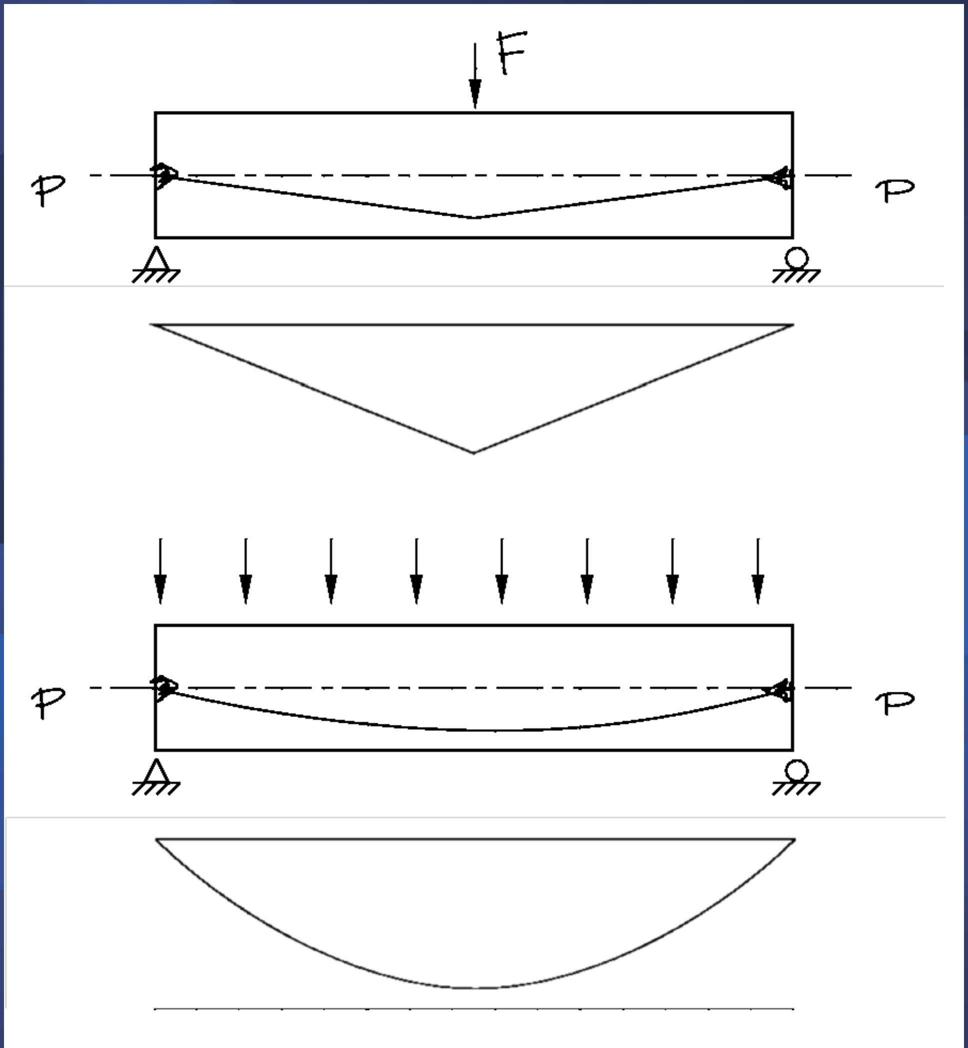


ECCENTRIC PRESTRESSING STRESSES AT SUPPORT



$$\frac{M_{max}}{S_{xx}} + \frac{P}{A} + \frac{P \cdot e}{S_{xx}} = f_c$$

VARY TENDON ECCENTRICITY



Harped Tendon follows moment diagram from concentrated load

Parabolic Drape follows moment diagram from uniformly distributed load

(T)

STRESSES AT TRANSFER - MIDSPAN

$f_{pi} = 0.7 f_{pu}$ $\quad f_{pi} = 189\,ksi$ Effective prestress in tendon

$P_i = f_{pi} \cdot A_{ps}$ $\quad P_i = 810 \cdot kip$ Including friction and elastic losses

$e_c = 21.19 \cdot in$

$$M_{max} = \frac{w_{sw} \cdot L^2}{8} \qquad M_{max} = 1192 \cdot kip \cdot ft$$

$$f_{top} = \frac{P_i}{A} - \frac{P_i \cdot e_c}{S_t} + \frac{M_{max}}{S_t} \qquad f_{top} = 445 \cdot psi$$

Compression

$$f_{bott} = \frac{P_i}{A} + \frac{P_i \cdot e_c}{S_b} - \frac{M_{max}}{S_b} \qquad f_{bott} = 1183 \cdot psi$$

Compression

$6 \cdot \sqrt{f'_{ci} \cdot psi} = 379\,psi$ tension at "end" of member

$3 \cdot \sqrt{f'_{ci} \cdot psi} = 190\,psi$ tension at other locations

$0.7 \cdot f'_{ci} = 2800\,psi$ compression stress limit at "end" of member

$0.6 \cdot f'_{ci} = 2400\,psi$ compression stress limit at other locations OK

STRESSES AT TRANSFER – FULL LENGTH

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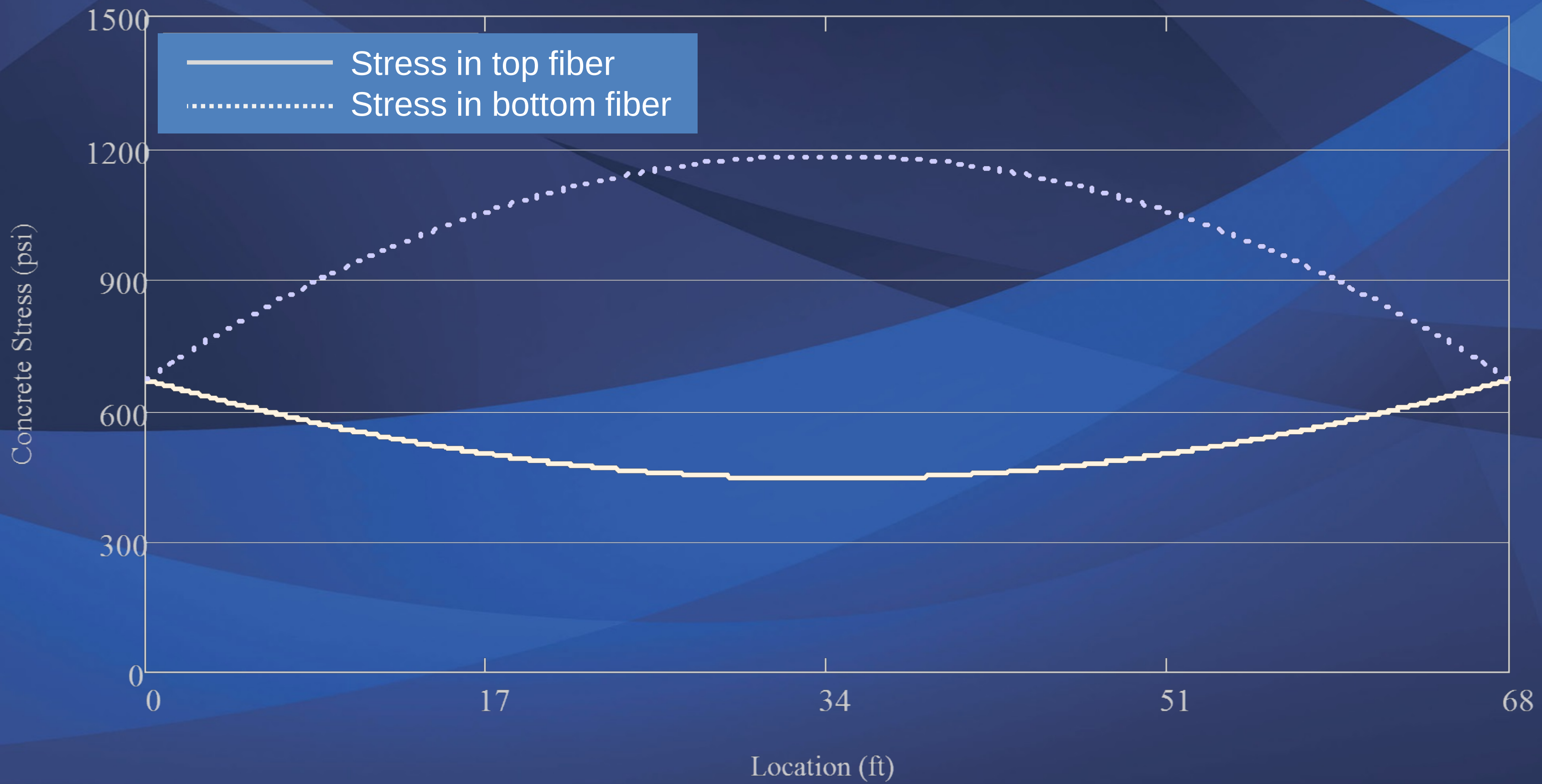


STRESSES AT SERVICE - MIDSPAN

$f_{se} = 175 \cdot ksi$ Effective prestress in tendon — Including all short and long term losses

$P_e = f_{se} \cdot A_{ps}$ $P_e = 750 \cdot kip$

$e_c = 21.19 \cdot in$

$$M_{max} = \frac{w \cdot L^2}{8} \qquad M_{max} = 1770 \cdot kip \cdot ft$$

$$f_{top} = \frac{P_e}{A} - \frac{P_e \cdot e_c}{S_t} + \frac{M_{max}}{S_t} \qquad f_{top} = 1047 \cdot psi$$ Compression

$$f_{bott} = \frac{P_e}{A} + \frac{P_e \cdot e_c}{S_b} - \frac{M_{max}}{S_b} \qquad f_{bott} = -338 \cdot psi$$ Tension

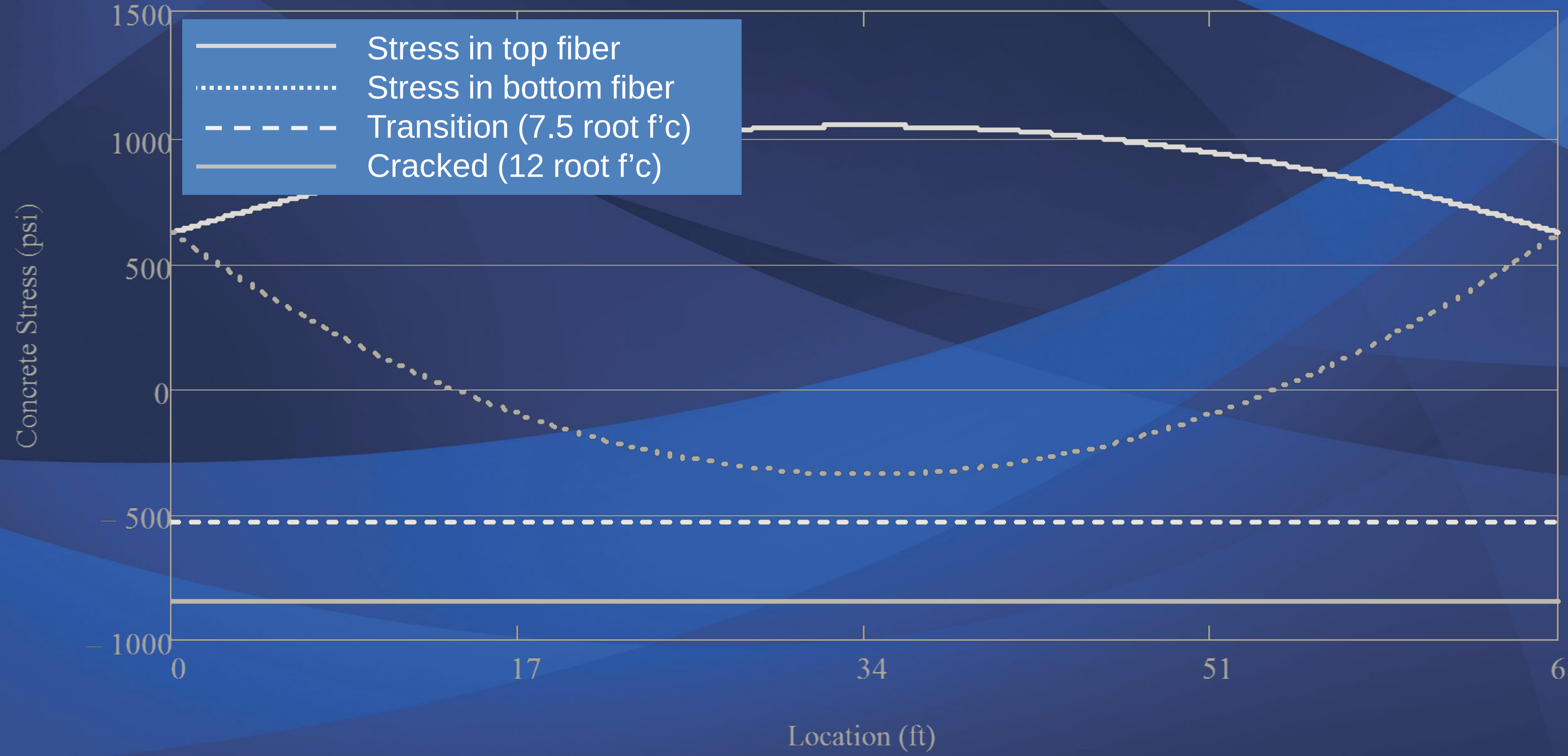
$7.5 \cdot \sqrt{f'_c \cdot psi} = 530 psi$ limit to be considered uncracked (Class U)

$12 \cdot \sqrt{f'_c \cdot psi} = 849 psi$ limit to be considered transition (Class T) — Class U member

$0.45 \cdot f'_c = 2250 psi$ compression stress limit for sustained loads plus prestress

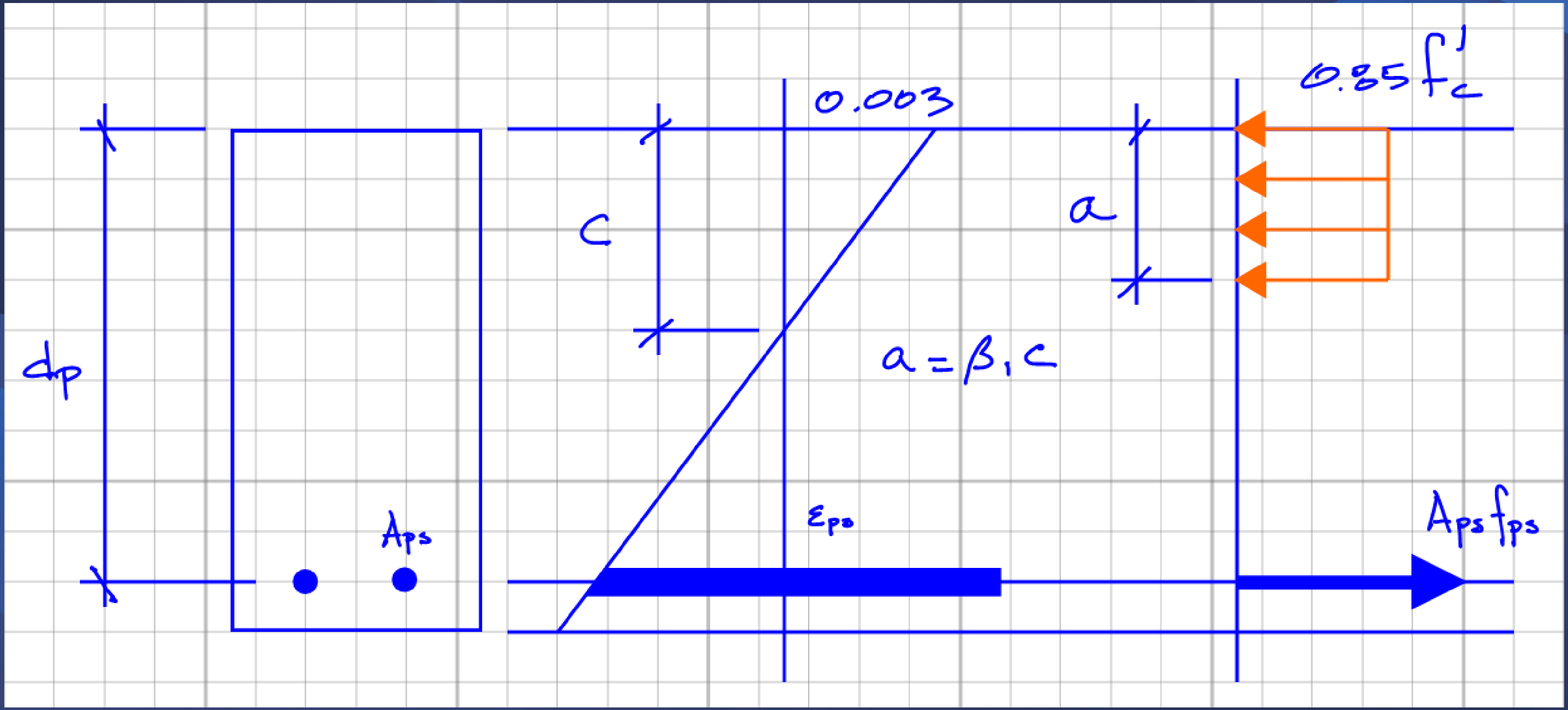
$0.6 \cdot f'_c = 3000 psi$ compression stress limit for total load plus prestress — OK

STRESSES AT SERVICE – FULL LENGTH



FLEXURAL STRENGTH (MN)

ACI 318 indicates that the design moment strength of flexural members are to be computed by the strength design procedure used for reinforced concrete with f_{ps} is substituted for f_y



ASSUMPTIONS

- Concrete strain capacity = 0.003
- Tension concrete ignored
- Equivalent stress block for concrete compression
- Strain diagram linear
- Mild steel: elastic perfectly plastic
- Prestressing steel: strain compatibility, or empirical
- Perfect bond (for bonded tendons)

f_{ps} - STRESS IN PRESTRESSING STEEL AT NOMINAL FLEXURAL STRENGTH

- Empirical (bonded and unbonded tendons)
- Strain compatibility (bonded only)

EMPIRICAL – BONDED TENDONS

$$f_{ps} = f_{pu}\left\{1 - \frac{\gamma_p}{\beta_1}\left[\rho_p \frac{f_{pu}}{f_c'} + \frac{d}{d_p}(\omega - \omega')\right]\right\} \qquad (18\text{-}3)$$

where ω is $\rho f_y / f_c'$, ω' is $\rho' f_y / f_c'$, and γ_p is 0.55 for f_{py}/f_{pu} not less than 0.80; 0.40 for f_{py}/f_{pu} not less than 0.85; and 0.28 for f_{py}/f_{pu} not less than 0.90.

270 ksi prestressing strand

If any compression reinforcement is taken into account when calculating f_{ps} by Eq. (18-3), the term

$$\left[\rho_p \frac{f_{pu}}{f_c'} + \frac{d}{d_p}(\omega - \omega')\right]$$

shall be taken not less than 0.17 and d' shall be no greater than $0.15 d_p$.

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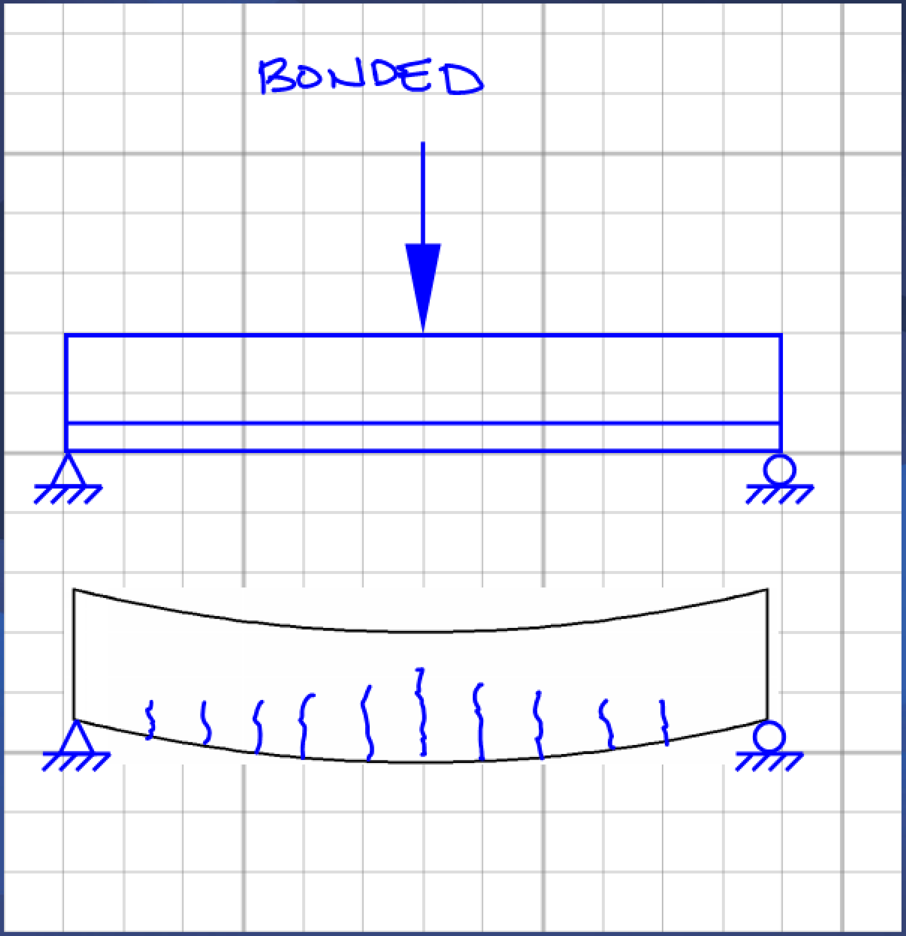
EMPIRICAL – BONDED TENDONS NO MILD STEEL

$$f_{ps} = f_{pu}\left\{1 - \frac{\gamma_p}{\beta_1}\left[\rho_p \frac{f_{pu}}{f_c'}\right]\right\}$$

ρ_p = ratio of A_{ps} to bd_p

BONDED VS. UNBONDED SYSTEMS



- Steel-Concrete force transfer is uniform along the length
- Assume steel strain = concrete strain (i.e. strain compatibility)
- Cracks restrained locally by steel bonded to adjacent concrete

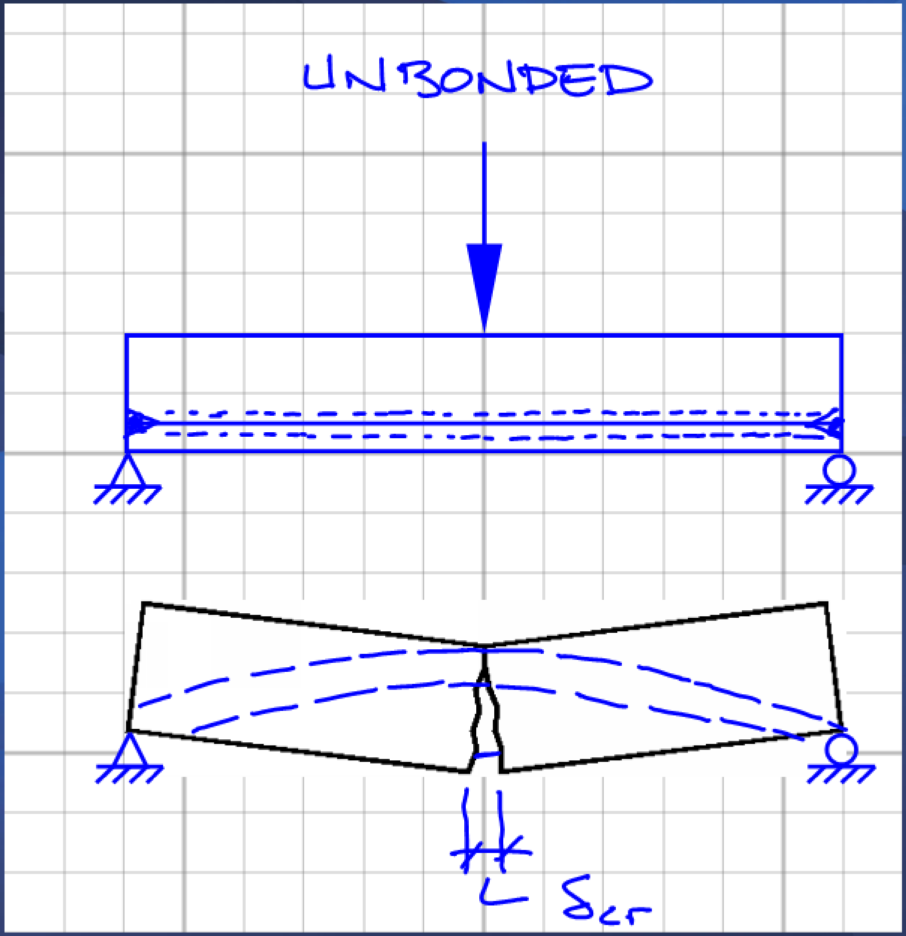
BONDED VS UNBONDED SYSTEMS

- Steel-Concrete force transfer occurs at anchor locations

- Strain compatibility cannot be assumed at all sections

- Cracks restrained globally by steel strain over the entire tendon length

- If sufficient mild reinforcement is not provided, large cracks are possible



SPAN-TO-DEPTH 35 OR LESS

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{100\rho_p} \qquad (18\text{-}4)$$

but f_{ps} in Eq. (18-4) shall not be taken greater than the lesser of f_{py} and **(f_{se} + 60,000)**.

SPAN-TO-DEPTH > 35

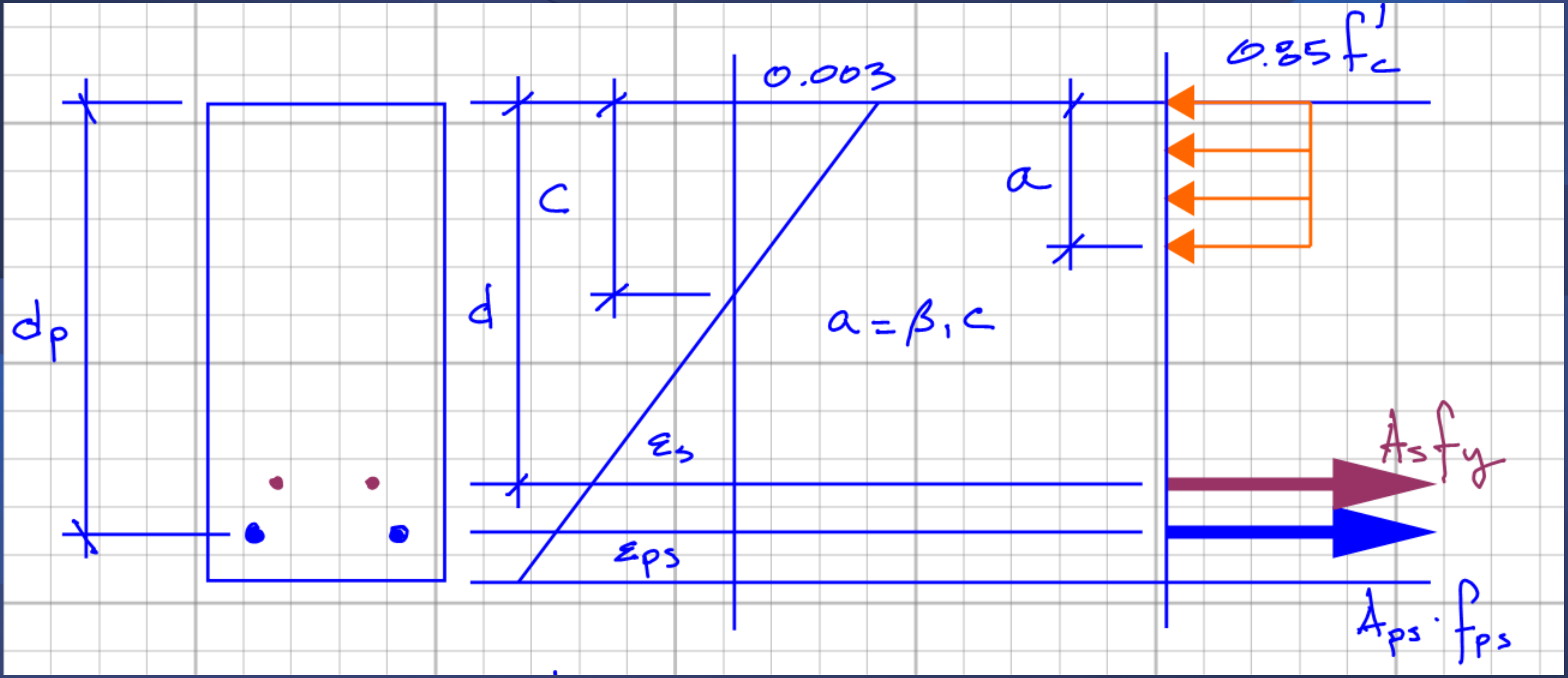
$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{300\rho_p} \qquad (18\text{-}5)$$

but f_{ps} in Eq. (18-5) shall not be taken greater than the lesser of f_{py} and **(f_{se} + 30,000)**.

Careful with units for f_{se} (psi)

COMBINED PRESTRESSING AND MILD STEEL

- Assume mild steel stress = fy
- Both tension forces contribute to Mn



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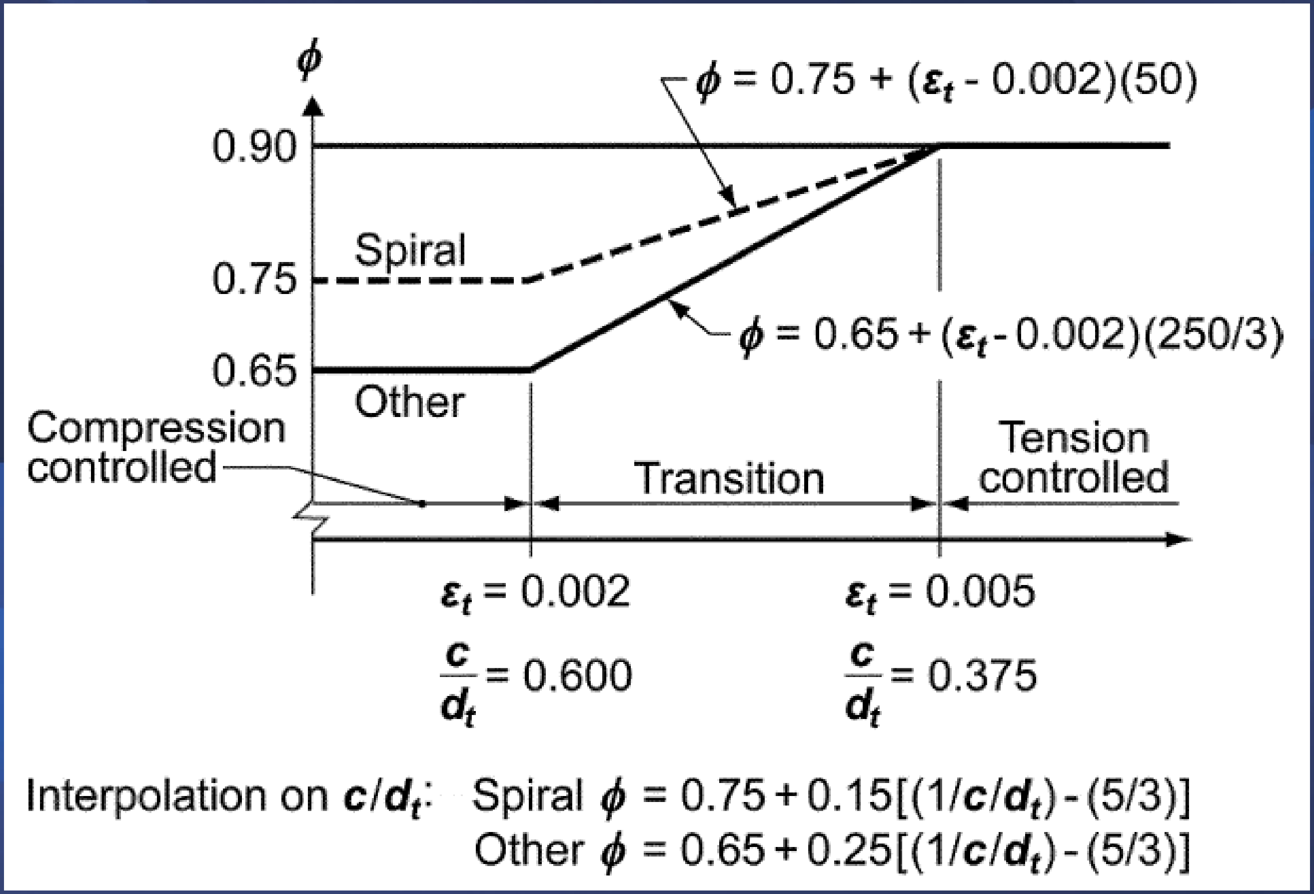
STRENGTH REDUCTION FACTOR ϕ

- Applied to nominal moment strength (M_n) to obtain design strength ($\phi\, M_n$)
- ranges from 0.6 to 0.9
- Determined from strain in extreme tension steel (mild or prestressing)
- Section is defined as compression controlled, transition, or tension controlled

STRENGTH REDUCTION FACTOR ϕ



DETERMINE FLEXURAL STRENGTH

- Is effective prestress sufficient?
- Determine f_{ps}
- Use equilibrium to determine:
  - Depth of stress block a
  - Nominal moment strength M_n
- Determine depth of neutral axis and strain in outside layer of steel (et)
- Determine ϕ
- Compute ϕM_n

f_{ps} OF BONDED TENDON

$\gamma_p = 0.28$

fpy/fpu > 90 for seven-wire prestressing strand

$d_p = 32.3 \text{ in}$

$A_s = 0 \quad A'_s = 0 \quad d = 0$

assume no mild reinforcement present. Use simplified version of fps equation.

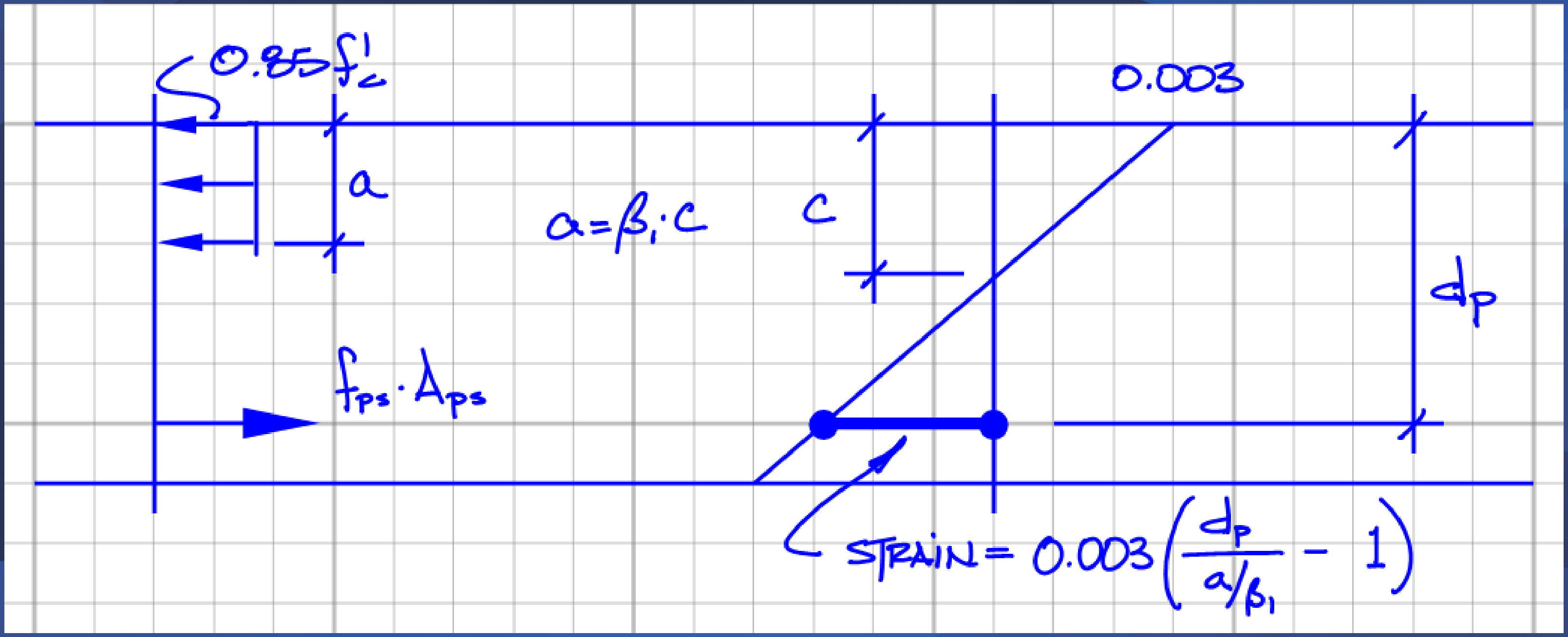
$$\rho_p = \frac{A_{ps}}{b \cdot d_p}$$

$\rho_p = 0.00120$ no change from unbonded

$$fps = f_{pu} \cdot \left[1 - \frac{\gamma_p}{\beta_1}\left(\rho_p \cdot \frac{f_{pu}}{f'_c}\right)\right]$$

$fps = \boxed{264 \cdot ksi}$ more than fps for unbonded

a and ϕ



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$$a = \frac{fps \cdot A_{ps}}{0.85 \cdot f'_c \cdot b} \qquad a = 2.40 \cdot in \qquad hf = 6 \cdot in$$

Check that the depth of the stress block is less than the thickness of the hollow core flange. OK.

$$strain = 0.003 \cdot \left(\frac{\beta_1 \cdot d_p}{a} - 1 \right) \qquad strain = 0.0293$$

Strain in the prestressing is greater than 0.005 so the phi factor is set equal to 0.9 (ACI 18.8.1)

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ϕM_N – BONDED TENDON

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$$\phi Mn = 0.9 \cdot fps \cdot A_{ps} \cdot \left(d_p - \frac{a}{2} \right)$$ sum moments about resultant compressive force

$$\phi Mn = 2633 \cdot kip \cdot ft$$

–

$$wu = 1.2 \cdot wDL + 1.2 \cdot w_{0p} + 1.6 \cdot wLL \qquad wu = 3.99 \cdot klf$$ Factored uniform load

$$Mu = \frac{1}{8} \cdot wu \cdot L^2 \qquad Mu = 2309 \cdot kip \cdot ft$$ Design moment strength is OK

REINFORCEMENT LIMITS

- Members containing bonded tendons must have sufficient flexural strength to avoid abrupt failure that might be precipitated by cracking.
- Members with unbonded tendons are not required to satisfy this provision.

REINFORCEMENT LIMITS

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$$f_r = 7.5\sqrt{f'_c \cdot psi}$$

$$Mcr = S_b \cdot \left(\frac{P_e}{A} + \frac{P_e \cdot e_c}{S_b} + f_r\right)$$

$$1.2 \cdot Mcr = 2231 \cdot kip \cdot ft$$

$$\phi Mn = 2633 \cdot kip \cdot ft$$

Cracking is considereed to have occurred when the net tensile stress exceeds the modulus of rupture. This occurs when the effective precompression and tensile strength are exceeded.

Flexural capacity exceeds 1.2 times the cracking moment. OK.

f_{ps} – UNBONDED TENDON

$0.5 f_{pu} = 135\,\text{ksi}$ $f_{se} = 175\,\text{ksi}$ effective prestress is sufficient to allow use of empirical eqn.

$\beta_1 = 0.8$ rectangular stress block factor. relates depth of stress block to depth of NA

$d_p = y_t + e_c$ $d_p = 32.3 \cdot \text{in}$ effective depth of prestressing steel

$$\rho_p = \frac{A_{ps}}{b \cdot d_p} \qquad \rho_p = 0.00120$$

$\frac{L}{h} = 22.7$ span-to-depth is less than 35. Use ACI eqn 18-4

$$f_{se} + 10\text{ksi} + \frac{f'_c}{100 \cdot \rho_p} = \boxed{227 \cdot \text{ksi}}$$

$$f_{se} + 60\text{ksi} = 235 \cdot \text{ksi}$$

$f_{py} = 0.9 \cdot f_{pu}$ $f_{py} = 243 \cdot \text{ksi}$ Equation 18-4 controls effective prestress at strength

ϕM_n – UNBONDED TENDON

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$a = \frac{fps \cdot A_{ps}}{0.85 \cdot f'_c \cdot b}$ $a = 2.06 \cdot in$ $hf = 6 \cdot in$ rectangular assumption OK

$strain = 0.003 \cdot \left(\frac{\beta_1 \cdot d_p}{a} - 1 \right)$ $strain = 0.0346$ strain > 0.005 phi factor is set equal to 0.9

$\phi Mn = 0.9 \cdot fps \cdot A_{ps} \cdot \left(d_p - \frac{a}{2} \right)$ $\phi Mn = 2275 \cdot kip \cdot ft$

$Mu = 2309\, kip \cdot ft$ not quite sufficient. Consider minimum bonded steel

MIN. BONDED REINF.

- Members with unbonded tendons must have a minimum area of bonded reinf.
- Must be placed as close to the tension face (precompressed tensile zone) as possible.
- $A_s = 0.004\ A_{ct}$
- A_{ct} – area of section in tension

AS MINIMUM

-

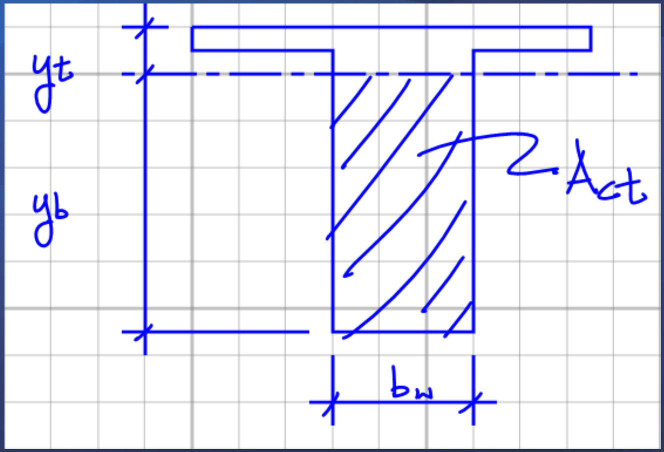
$$Act = y_b \cdot bw \qquad Act = 448.9 \cdot in^2$$

$$AsMin = 0.004 \cdot Act \qquad AsMin = 1.80 \cdot in^2$$

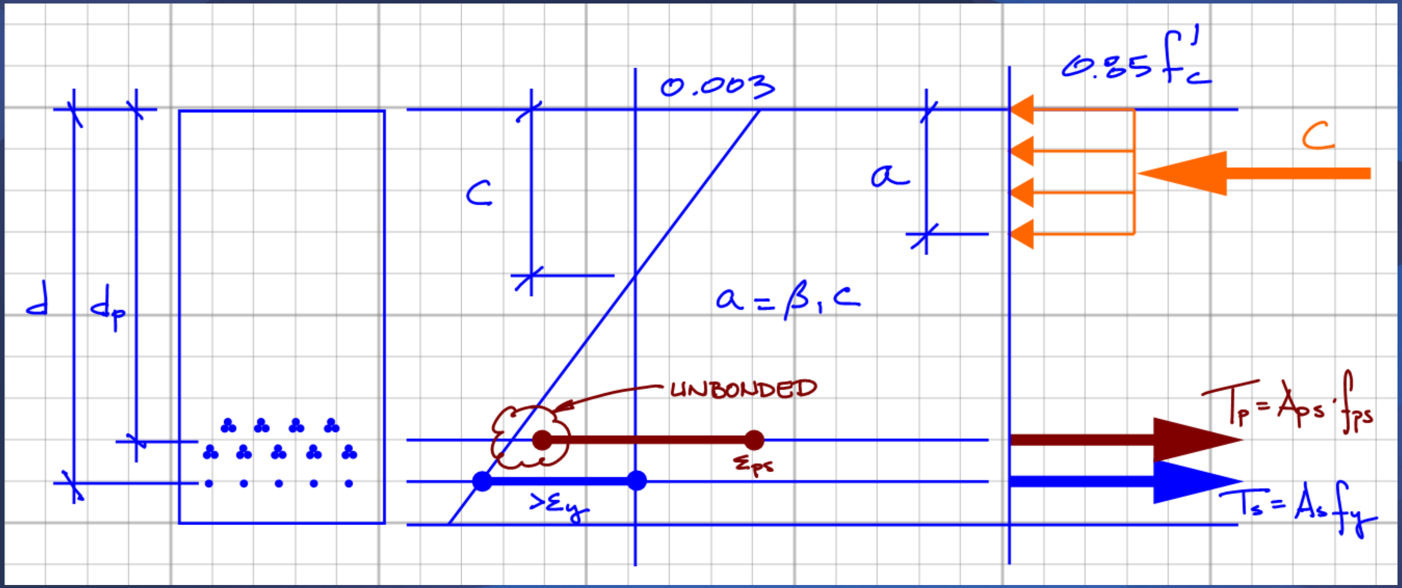
$$SixNo5 = 6As5 \qquad SixNo5 = 1.86 \cdot in^2$$

Add six #5 bars to flexural tension face of beam

$$d = h - 1.5in - db3 - 0.5db5 \qquad d = 33.8 \cdot in$$



ϕM_n – UNBONDED TENDON INCORPORATE MILD STEEL



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$$a = \frac{fps \cdot A_{ps} + f_y \cdot AsMin}{0.85 \cdot f'_c \cdot b} \qquad a = 2.29 \cdot in \qquad hf = 6 \cdot in$$

$$strain = 0.003 \cdot \left(\frac{\beta_1 \cdot d}{a} - 1 \right) \qquad strain = 0.0325$$

$$\phi Mn = 0.9 \cdot \left[fps \cdot A_{ps} \cdot \left(d_p - \frac{a}{2} \right) + f_y \cdot AsMin \cdot \left(d - \frac{a}{2} \right) \right] \qquad \phi Mn = 2531 \cdot kip \cdot ft$$