

DESIGN OF CONCRETE FLOORS With Particular Reference to Post-Tensioning

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ABSTRACT

This paper presents the key features of concrete design and highlights its differences from design of other materials. The three commonly used analysis methods for concrete floors: Simple Frame, Equivalent Frame and Finite Elements are discussed. It is demonstrated that regardless of the method of analysis, designation of a load path is a prerequisite for concrete floor design. Integration of actions over design strips is presented as a fundamental step in the interpretation of the solution when using the Finite Element method. Design characteristics of post-tensioned floor systems are outlined and the importance of interpreting representative (hypothetical) stresses for building code compliance is emphasized. The paper concludes with a presentation of a structural modeling option for the analysis of waffle slabs.

1. FEATURES OF CONCRETE DESIGN

1.1 General Requirements

The primary concerns of a structural engineer are the safety, serviceability, and economy of the structures he or she designs. Safety is understood as the structure's ability to withstand code required loads without excessive damage. Serviceability is achieved if the structure performs as intended throughout its expected life span. Economy is taken to mean the structure's owners feel that both its short and long-term costs are reasonable.

Legality of the design procedure, defined as compliance with applicable building codes, is also important. It is not always easy to establish however, particularly for post-tensioned structures. Codes tend to follow, rather than lead, practice with respect to post-tensioning. Much of what is currently considered appropriate practice for post-tensioned design is not yet incorporated into the codes.

1.2 Requirements of Design Procedure

An important issue which is often overlooked by code developers is that unless design procedures are fairly simple and expedient, they will not be adopted by design professionals. This can be important when a design professional is asked to evaluate and select from different structural alternatives.

An example within the field of concrete design is the use of post-tensioning in building construction. Some consulting engineers are reluctant to select a post-tensioned alternate because the design may require more time and effort than a conventional concrete structure. Although the post-tensioned structure may be more economical and have superior performance, the engineer typically cannot increase his or her fee. Consequently, some design engineers have developed an aversion to post-tensioning and seek reasons not to use it.

Another, perhaps more important, issue facing the concrete industry in many countries is the role played by the design professional in the selection of construction materials, namely the choice between concrete and steel. In the recent past, the advent of computers and automation in design and detailing has been favorable to the steel industry. Steel framing benefited from an initial advantage in that it readily lent itself to design automation. There are now computer programs that can perform a complete steel design, including detailing, with minimal input from the design professional. This has not been true for concrete buildings; integrated computer programs have been lacking. In addition, the concrete design codes have become more complex and require greater input from the design professional.

The difficult part of automating concrete design is the floor slab. In most cases, skeletal members such as beams, columns, and frames

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made from them can be readily analyzed and designed. Concrete buildings are rarely limited to skeletal members, however, the floor slabs are typically a significant portion of the building and its design.

This article reviews concrete slab design concepts and presents a method for automating the design of both reinforced and post-tensioned concrete buildings. It is anticipated that this method will eliminate many of the problems associated with the integrated design of concrete buildings. A procedure for selecting load paths and guidelines for the layout of reinforcement are also presented.

1.3 Concrete Design in Relation to Other Materials

The following example highlights the principal features that distinguish concrete design from other types of design. The example considers three materials: concrete, steel and glass, each of which has a distinctive feature in terms of design. Although the example is hypothetical, it illustrates how material properties affect design requirements and procedures.

Fig. 1.3-1(a) shows a partial plan of a plate or slab under uniform loading. The example reviews the design of the area surrounded by supports A, B and C, marked "Design Region." The objective is to satisfy the serviceability and safety (strength) requirements of this region.

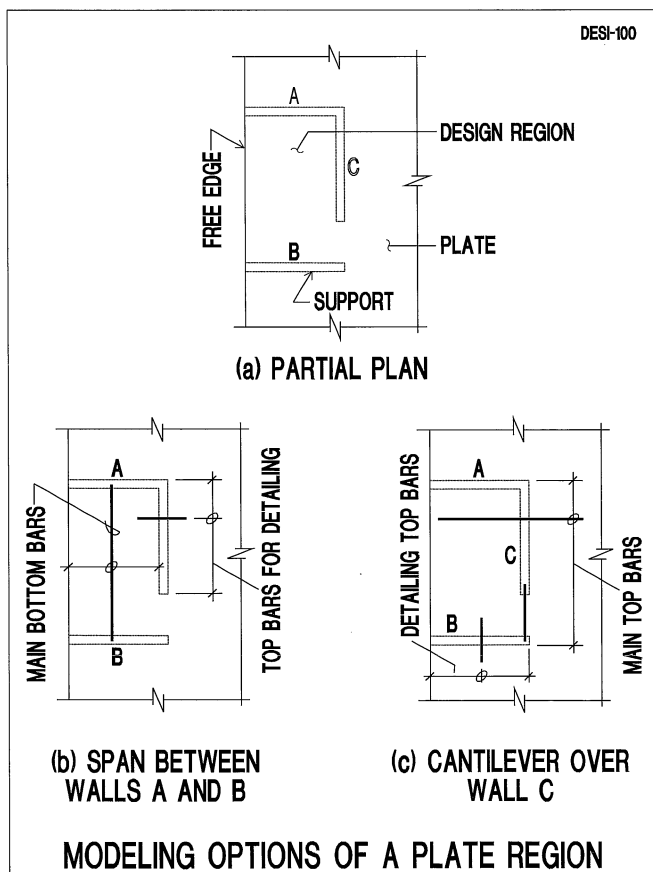


FIGURE 1.3-1

A. Glass

Consider first a glass plate. The serviceability of the glass plate is determined by acceptable deflection; its safety is measured by the load that causes it to crack. Cracking occurs when the tensile stress at the surface reaches a value that is a material property of the glass. Glass is an extremely brittle material; once cracking is initiated, it will spread immediately and cause failure. Hence, the design procedure consists of:

- Estimating the deflection under service load; and
- Determining the load at which the maximum tensile stress reaches the cracking strength of glass.

For serviceability design, deflections can be estimated using approximate methods based on the plate's geometry, support conditions, material properties and service loading. As noted above, however, failure occurs when the stress at any point on the plate reaches the cracking strength of the glass. In order to get a reliable estimate of the glass plate's safety, both the location of the maximum stress and the stress value in relation to the applied loading need to be determined accurately.

The geometry and supports of the glass plate must be modeled accurately since they directly affect the magnitude of maximum stress. In most cases, the actual load path must be determined either analytically or experimentally. Approximate methods based on assumed load paths will not produce accurate results.

The need to accurately determine the stress at a point in order to ensure the safety of glass plate is what differentiates the response of glass under increasing load from that of the other materials. Local stresses calculated by finite element analysis are sensitive to the number of mesh divisions and the accuracy of the finite element formulation. In order to determine the value of stress at point, a very fine mesh and an appropriate formulation must be used.

B. Steel

Assuming that the design region in Fig. 1.3-1(a) is made out of steel plate, its serviceability is governed by its deflection and permanent deformation under service loading. Its safety or ultimate strength is generally determined by excessive deformation under factored loading.

Although approximate modeling can be used to estimate the deformation of the plate under service loading, there will be permanent deformation if local yielding occurs. Local yielding is a function of the distribution of stress under service loading. Using von Mises criterion, yielding occurs when the combination of stresses at a point reaches a characteristic value of the material. Either experimental techniques or a finely meshed finite element analysis must be used to evaluate local yielding. The reliability of

the design depends on the accuracy with which the location and magnitude of the local stresses are calculated.

The design procedure is as follows:

- Assume an initial plate thickness and support conditions;
- Estimate deflection under service load using approximate methods;
- Use a rigorous method to determine local stresses and the likelihood of local yielding under service loads in order to avoid permanent deformation; and
- Determine the strength (safety) limit by using an approximate method to estimate the load at which overall plastification results in excessive deformation. (A rigorous method can be used for this but is generally not warranted.)

A central feature of steel design is that there is typically an initial assumption of plate thickness and supports conditions. The calculations are then aimed at verifying the location and magnitude of the maximum von Mises stress. If the calculated stresses are less than allowable stress limits, the initial assumptions are regarded as an acceptable design. The fact that the design is essentially a verification of the initial assumptions is one of the key features which differentiates steel design from concrete design.

C. Concrete

Consider finally a concrete plate. Design criteria for the concrete plate are: (i) the deflections and crack widths must be within acceptable limits under service conditions, and (ii) the plate must not collapse under code stipulated factored loads.

Determination of local stresses is generally not meaningful when evaluating deflections and crack widths of concrete under service loading. Microcracking and lack of material homogeneity make the use of simple linear-elastic analysis invalid; rigorous analysis is typically not warranted.

Moreover, unlike the design procedure for glass, the determination of local stresses does not need to be exact. A typical slab is usually modeled and designed following designated load paths as opposed to the analytically determined load paths used for the glass and steel plates.

The ability of a concrete section to crack and undergo a finite amount of rotation prior to failure is a reflection of the section's ductility. In order to mobilize the assumed load paths and redistribute the load resistance in a floor, the slab must possess a certain ductility. ACI-318 ensures a minimum ductility by limiting the amount of reinforcement in a section; this effectively limits the depth to the neutral axis. Other major codes impose similar limits.

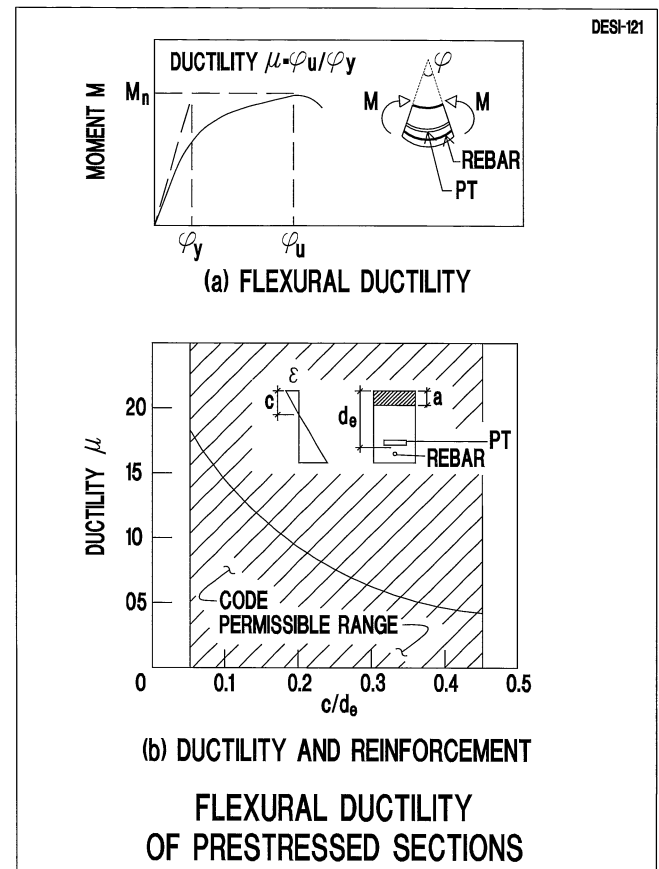


FIGURE 1.3-2

Fig. 1.3-2 is a schematic of a typical region of a post-tensioned beam designed according to ACI-318. The ratio of the ultimate curvature, ϕ_u , to the curvature at first yield, ϕ_y , gives a measure of the ductility of the section. Sections which comply with code restrictions on reinforcement and depth to the neutral axis are likely to have a ductility ratio in the range of 4 to 18.

For the example in Fig. 1.3-1(a), two load path options are shown (Fig. 1.3-1(b) and 1.3-1(c)). In Fig. 1.3-1(b) the slab is modeled as a strip spanning between walls A and B. In other words, the engineer designates a "load path" for the transfer of the design load to the two supports. For this load path, the reinforcement for safety against collapse will be bottom bars, referred to as primary reinforcement, between walls A and B.

Wall C is not a designated part of the load path shown in Fig. 1.3-1(b). Nevertheless, it will participate in supporting the load; the slab over it is thus likely to develop high tensile stresses. The design engineer must recognize this and place a nominal amount of top bars over the wall for crack control under service loading. This process of adding steel in selected locations is referred to as "Structural Detailing". Structural Detailing is an essential step in concrete design and is highly dependent on the experience and engineering judgment of the design professional.

Structural Detailing is done after the amount and location of the primary reinforcement is determined. It fulfills the design concept by ensuring that:

- The load path envisaged by the engineer can develop at loadings equal to, or greater than, code stipulated values;
- The crack widths under service loading are within acceptable limits.

Referring back to the original plate design, in Fig. 1.3-1(c), the slab is modeled as a cantilever supported by wall C. This load path requires top bars over wall C as shown. These bars are supplemented by Structural Detailing top bars placed over walls A and B for crack control.

The detailing involved in translating the bottom bars shown in Fig. 1.3-3(a-i) into the number, length and location of bars to be placed in the slab as shown in (a-ii) is referred to as "Construction Detailing". Another example of Construction Detailing is the selection of the correct lap splices, hooks, and bar bending details as illustrated in Fig. 1.3-3(b).

In North America, Construction Detailing is shown on shop drawings generated by the materials suppliers. Structural Detailing, on the other hand, is done by the design engineer and is shown on the structural drawings. In many other parts of the world, however,

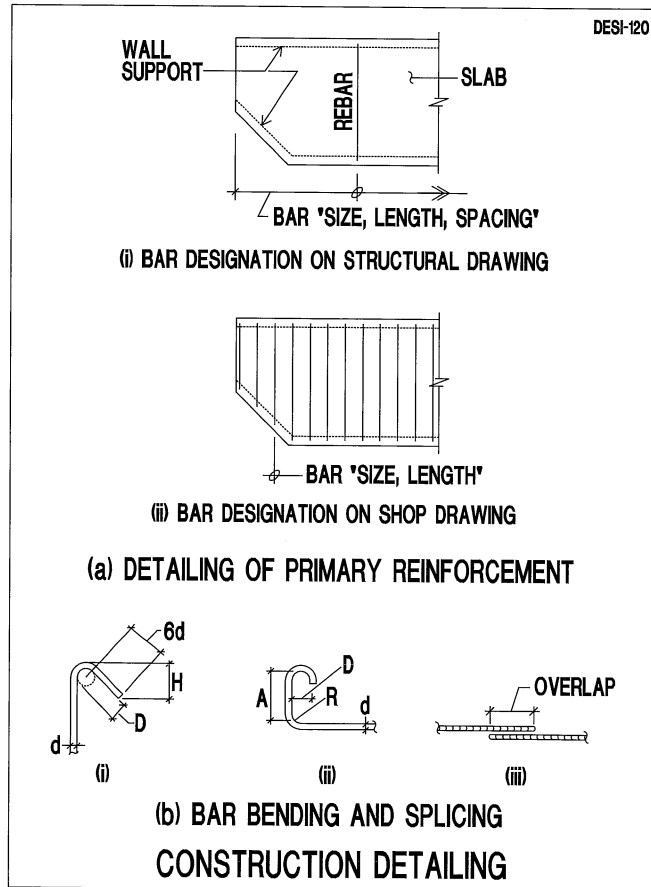


FIGURE 1.3-3

there is no distinction between Structural Detailing and Construction Detailing. Unlike the practice in North America, the drawings generated by the design engineer also reflect the Construction Detailing.

Fig. 1.3-4(a) shows an example of Structural Detailing for crack control. Fig. 1.3-4(b) shows an example of Structural Detailing for development of the load path. In Fig. 1.3-4(b), a concentrated loading is distributed over the width of the assumed load path by distribution steel placed underneath the load. The added reinforcement ensures that the load path between walls A and B is able to materialize as envisaged by the designer. Note, however, that although this type of reinforcement is required for both safety and serviceability, it is not reflected in many design methods, including the one introduced in this article.

The following general conclusions can be drawn about the design of concrete structures:

- When doing concrete design, the engineer must designate a load path in order to determine the reinforcement. This is unlike the glass alternative, where the load path must be determined by analysis, or the steel alternative, where local yielding can be determined without a designated load path. The load path designation is required for concrete because the layout of the reinforcing bars governs the orientation and

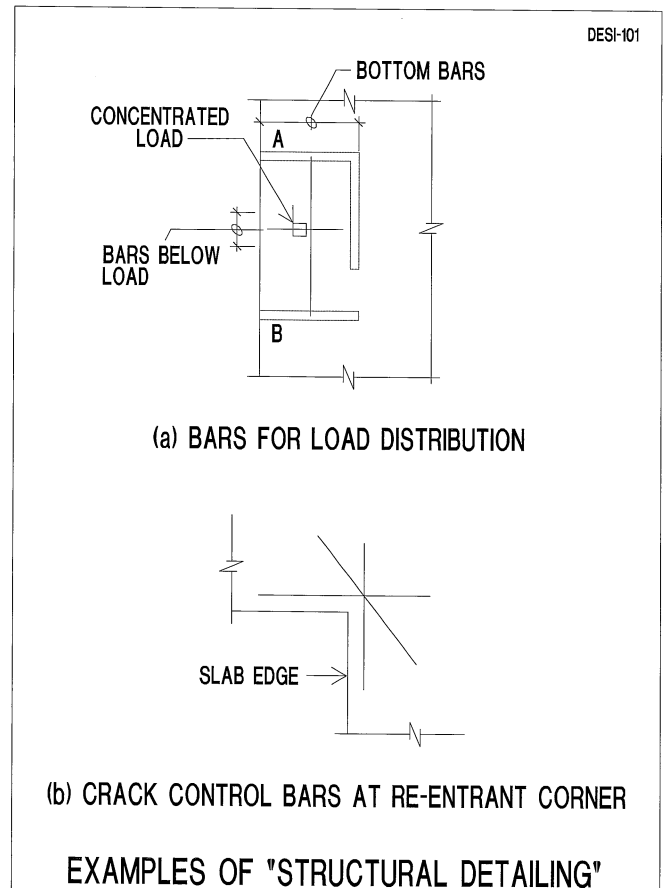


FIGURE 1.3-4

magnitude of the resistance developed by the slab. Often, there is more than one possible load path. The load paths that are selected make up the skeleton of the "structural system" of the building.

- Concrete design is not sensitive to local stresses. The distribution of moments determined from elastic theory will be similar to the schematics of Fig. 1.3-5(a). The simplified, equivalent moment shown in Fig. 1.3-5(b) is generally used for reinforcement calculations, however. The reinforcement necessary for strength in each direction is that required to resist the total moment, i.e. the integral below the moment curve. The layout of the reinforcement is typically not critical as long as the bars are within the region corresponding to the moment they are designed to resist. This is based on the premise that failure follows the formation of a hinge line and the hinge line will mobilize all of the reinforcement that crosses it.

This highlights another feature of concrete design which is that the total moment is used for design. The distribution of the moment and local values of the moment are not critical. The total moment is considered to be resisted by a "design section" as opposed to glass and steel design where local moments are checked at design points. This feature places concrete at a great computational advantage since total (integral) values of

the actions are not as sensitive to finite element discretization as local values. Finite element software is generally formulated to satisfy static equilibrium, regardless of the density of the mesh used to discretize the structure. A coarse mesh gives essentially the same "total" moment over a design section as a fine mesh. This observation is discussed further in [Aalami, Bommer., 1999].

- As with glass and steel, stresses in prestressed concrete structures must be checked under service load. With glass, the objective of the stress check is to avoid breakage; with steel, the objective is to avoid local yielding and permanent deformation. With concrete, however, the objective is generally to control - not avoid - crack formation.

The calculated concrete stresses used for design are "hypothetical," since they are based on the application of the total moment to the entire design section. In reality, stresses over the supports will peak to much higher values. In most instances they will exceed the cracking limit of the concrete. The calculated stresses are thus an indication of the extent of crack formation over a region rather than true stresses.

- Note that for one-way systems, the limit on these hypothetical tensile stresses in ACI 318 is one $\sqrt{f'_c}$ ($12\sqrt{f'_c}$ in inch-pound units) even though the cracking limit (modulus of rupture) of concrete is typically considered to be between 0.5 and 0.625 $\sqrt{f'_c}$ (6 to 7.5 $\sqrt{f'_c}$ in inch-pound units). The hypothetical tensile stress limits are even higher in many other building codes, i.e. the Canadian code [CAN 23.3, 1994].

1.4 Design Characteristic of Post-Tensioned Floors

The need to designate a "load path" was identified as one of the characteristics that differentiates concrete design from steel design. Post-tensioned concrete design adds another layer of complexity that requires additional engineering judgment and input. Consider the example of Fig. 1.3-1(b), in which the slab region is assumed to span between walls A and B. A post-tensioned design alternative of this region is shown in Fig. 1.4-1. The region is reinforced with post-tensioning tendons between walls A and B; the post-tensioning is supplemented by mild steel as shown.

The serviceability and safety of the design are controlled by two principal parameters: (i) the amount of prestressing, and (ii) the tendon profile (the distance between the center of the tendon and centroid of the slab along the tendon's length). Typically, the amount of mild reinforcement required will depend on what is selected for these parameters.

In a nonprestressed floor, determination of the required reinforcement is fairly routine once the load path is determined. The results in terms of required reinforcement will be essentially the same, regardless of

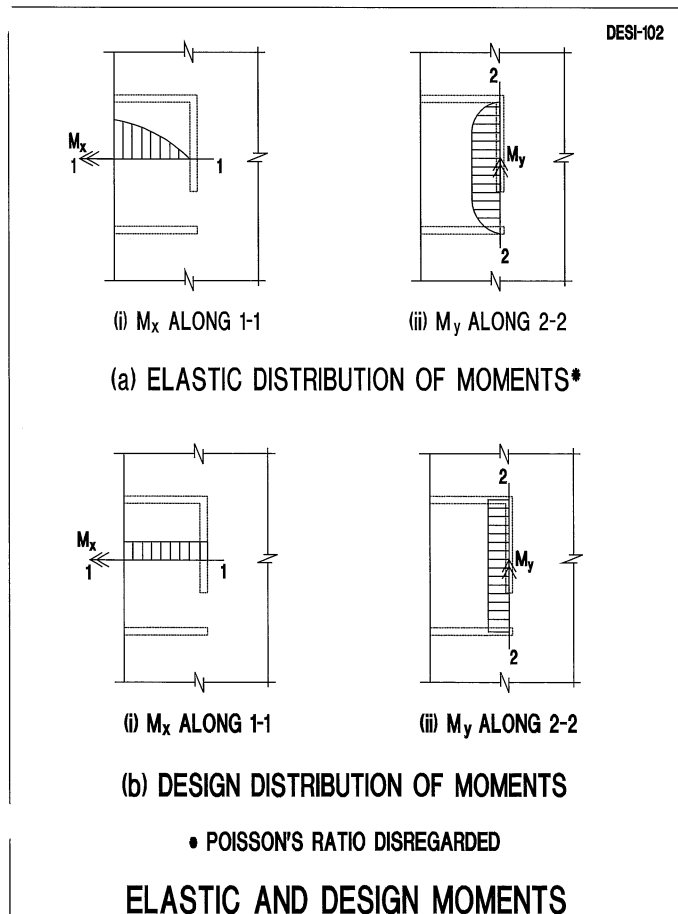


FIGURE 1.3-5

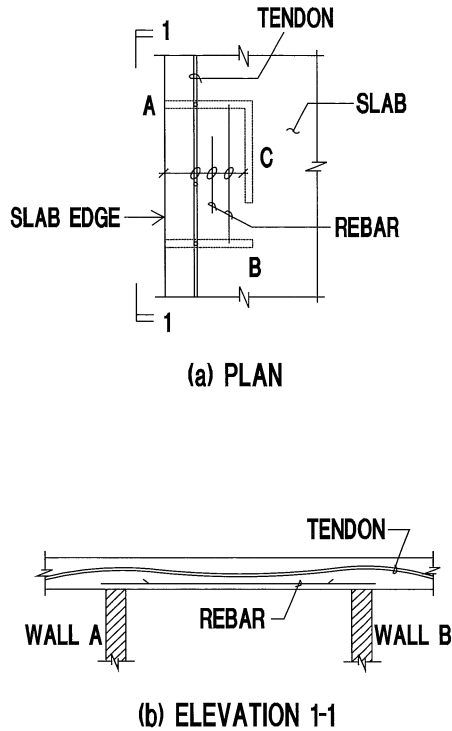


FIGURE 1.4-1

the designer's experience or inclination. In a post-tensioned floor, however, the engineer has considerable latitude when selecting the amount of prestressing and the tendon profiles. Depending on the engineer's assumptions and what he or she uses as design criteria, different designs will result. Automation of a post-tensioning design thus requires additional steps to select post-tensioning forces and tendon profiles.

1.5 Automation of Concrete Design

In summary, the design of concrete floors as practiced today requires a designated "load path" for the determination of primary reinforcement. If the slab is post-tensioned, additional assumptions are required in order to design the post-tensioning. In addition, in post-tensioned structures, stresses need to be checked under service loads. Unlike glass and steel however, the stress used in design of prestressed concrete is a "hypothetical" stress, when sections are cracked, and "smeared," or average stresses when the section is not cracked. The stress calculated for use in design is not necessarily the local stress. Consequently, it is not necessary to perform a finely meshed finite element analysis to determine local stresses.

Designation of the load path and determination of the primary reinforcement must be followed by Structural Detailing. To date, no attempts to automate either the designation of load paths or the

Structural Detailing functions have been reported in the literature. The following section presents a procedure which automates much of the design procedure for a nonprestressed floor and the initial steps of the design of a post-tensioned floor. The additional steps required for post-tensioned design are discussed in [Aalami, Kelley 2001].

2. DESIGN PROCESS

2.1 Structural Modeling and Analysis

The design process for concrete floors is summarized in the flow chart shown in Fig. 2.1-1. There are essentially four steps: a **structural modeling** step, an **analysis** step, a **design** step and a **Structural Detailing** step. The structural modeling step involves designating load paths. The analysis step determines the actions (moments and shears) that each load path must resist. The design step gives the area of reinforcement required to resist these actions. The Structural Detailing step determines the layout of the reinforcement; it also determines if additional steel is needed for crack control or load distribution.

2.1.1 Analysis Methods

The three methods commonly used for analysis of concrete structures are: **Simple Frame**, **Equivalent Frame**, and **Finite Element** analysis. In the Simple Frame method, the slab is divided

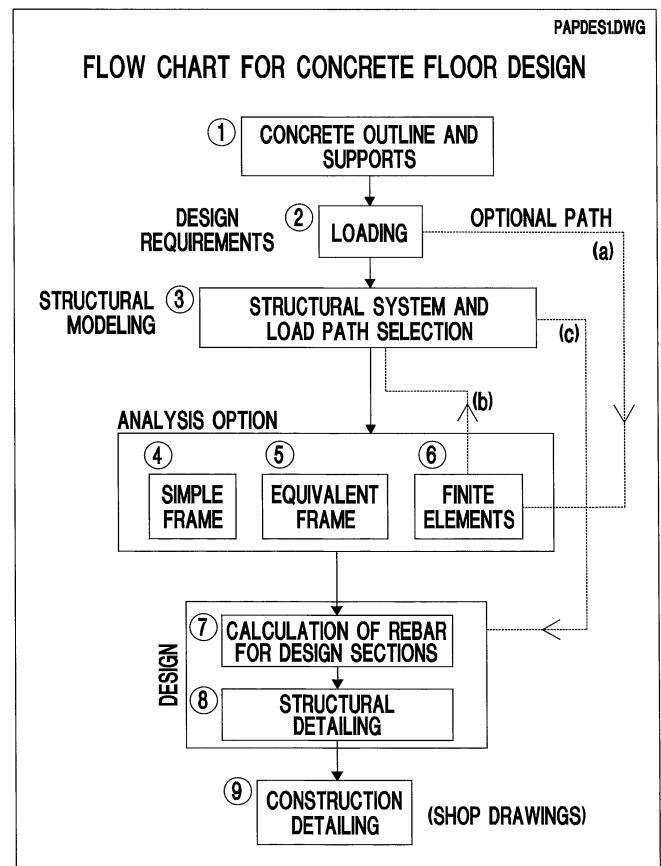


FIGURE 2.1-1

into design strips. The geometry of the structure is modeled exactly, i.e., the frames are analyzed using the stiffnesses of the columns and associated slabs as calculated from their geometries. As a result, the analysis does not account for the influence of biaxial plate bending.

The Equivalent Frame method (EFM) is a refinement of the Simple Frame method. It is somewhat more exact than the Simple Frame method since the relative column and slab stiffnesses are adjusted to account for the biaxial plate bending. In typical design, most column-supported floors are analyzed with the Equivalent Frame method.

The information required as far as geometry, loading and boundary conditions is the same for both the Simple Frame and the Equivalent Frame methods. Although both methods are approximate, they both yield lower bound (safe) solutions. The degree of approximation depends on the extent to which a floor system deviates from a uniform, orthogonal support layout and constant slab thickness.

The third method of analysis, the Finite Element method (FEM), is based on the division of the structure into small pieces (elements) whose behavior is formulated to capture the local behavior of the structure (Fig. 2.1.1-1). Each element's definition is based on its material properties, geometry, location in the structure, and relationship with surrounding elements. The mathematical assemblage of these elements into the complete structure allows for automated computation of the response of the entire structure. FEM inherently incorporates the biaxial behavior of the floor system when determining the actions in the floor.

The following references give a general description of the Finite Element method [Zienkiewicz, Taylor, 1989; Bathe, 1982]. Details of the procedure with specific application to concrete floor systems are given in [Aalami, Bommer, 1999].

2.2 Load Path Designation

The focus of the structural modeling step is the designation of the load paths. The structural system is complete if the self-weight and applied loading at every location is assigned an explicit load path to a support.

Load path designation is based on the strip method. This requires dividing the floor into intersecting support lines, each of which has its own tributary area. The support lines indicate the assumed load paths; a support line, together with its tributary area, is referred to as a "design strip." For nonprestressed floors, the load path is determined by the position and orientation of the reinforcement. For prestressed floors, the load path is determined by the layout of post-tensioning tendons.

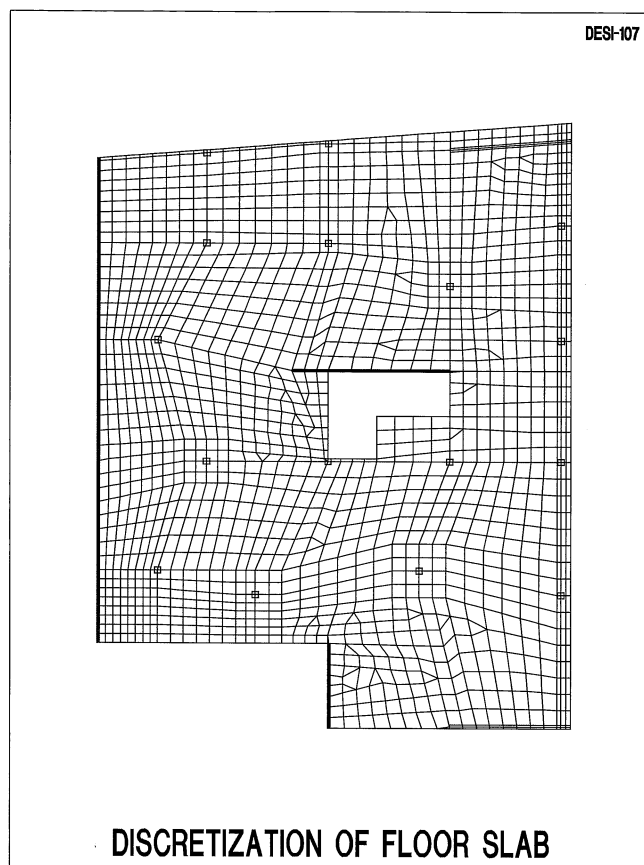


FIGURE 2.1.1-1

For most structures, selection of the load paths is essentially independent of the analysis method. Consider a typical floor from a multi-story building with columns and walls above and below as shown in Fig. 2.2-1. The following describes the structural modeling of the floor and illustrates the procedure for selecting load paths and design sections.

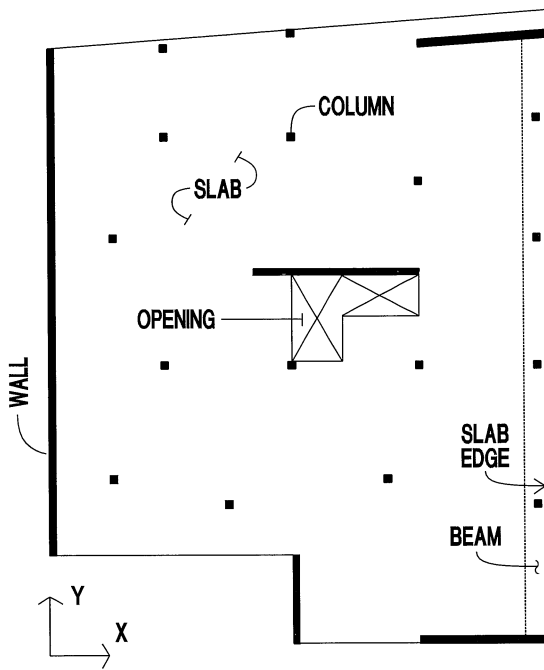
Define Outline of Floor Slab and Supports

As a first step, the engineer defines the slab edge and any openings, steps or other discontinuities. Next, he or she identifies the location and dimensions of the walls and columns supporting the floor. The supports for this example are shown in Fig. 2.2-1. Note that beams are considered as part of the floor system rather than the support system. They are therefore modeled and designed in conjunction with the floor slab.

Define Support Lines

The engineer then determines a series of support lines in each of the two principal directions. Typically, these are lines joining adjacent supports along which an experienced engineer will intuitively place reinforcement. Figure 2.2-2 shows the support lines, labeled A through G, in the X-direction (F is not a designated line of support). Fig. 2.2-3 shows the five support lines, labeled 1 through 5, in the Y-direction.

If a floor system is highly irregular, i.e. the columns are



PLAN OF FLOOR SLAB

FIGURE 2.2.1

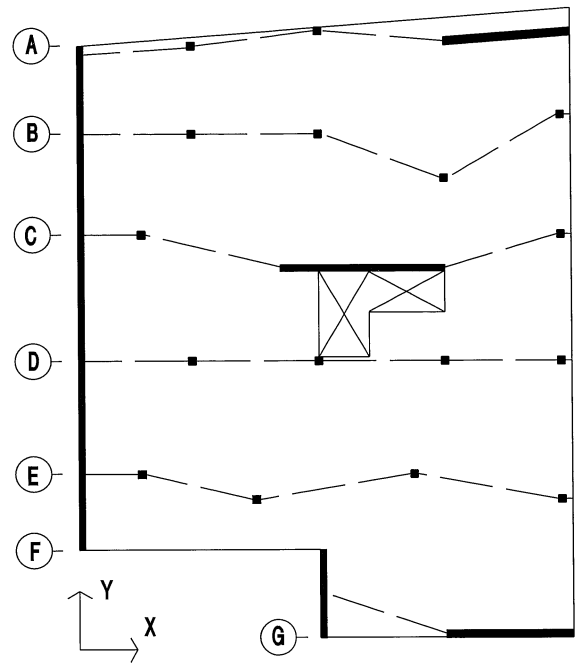
DESIGNATION OF LINES OF SUPPORT
IN X-DIRECTION

FIGURE 2.2-2

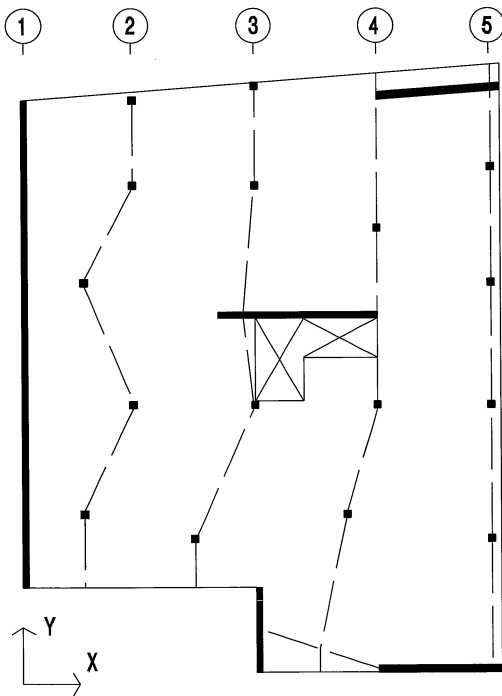
DESIGNATION OF LINES OF SUPPORT
IN Y-DIRECTION

FIGURE 2.2-3

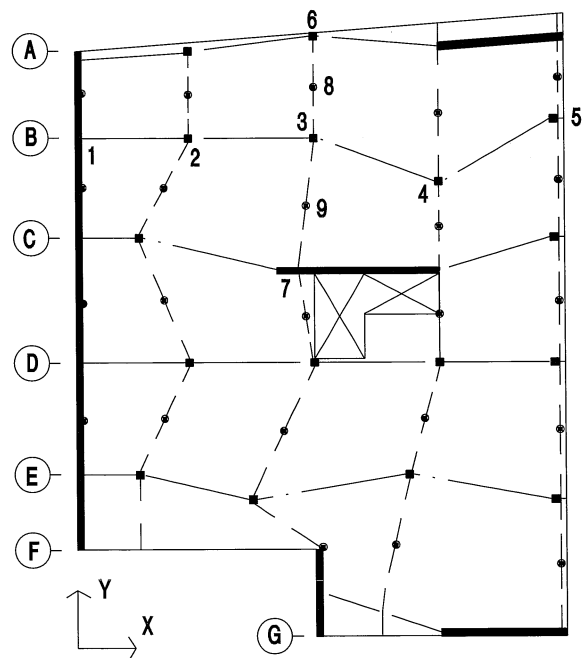
SELECTION OF DEMARCATION
POINTS FOR TRIBUTARIES (DESIGN STRIPS)

FIGURE 2.2-4

significantly offset from one another, the support lines may be less obvious. The criteria for selection are the same as in a regular slab, however. The support lines are the lines along which an experienced structural engineer is likely to place the primary reinforcement for resisting the gravity load.

Define Tributaries and Design Strips

Typically, the midpoints between support lines are used to designate the tributary areas for each support line. The midpoints are joined to identify the boundaries of the tributary. Figure 2.2-4 shows the support lines in the X-direction. Points 8 and 9 would be used to determine the boundaries for the tributary of support line B, for example. The tributaries for the design strips in the X-direction are hatched in Figure 2.2-5. Figure 2.2-6 shows the support lines in the Y-direction with their associated tributary areas.

Design Sections

Design sections are typically drawn across each design strip at the locations where the integrated actions on the design strip are greatest. There is no limit to the number of design sections that can be specified. Note that the maximum design actions in the field may not be at the midpoint of the spans. In addition, peak design actions for the strength and serviceability checks may not occur at the same location. Figure 2.2-7 shows the design sections for two of the design strips in the X-direction. Across the width of the

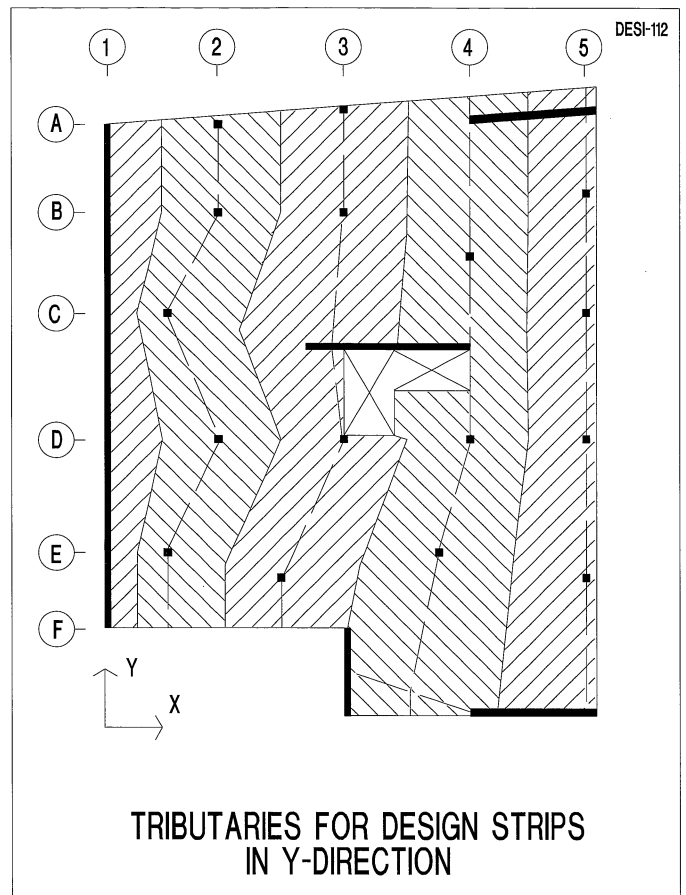


FIGURE 2.2-6

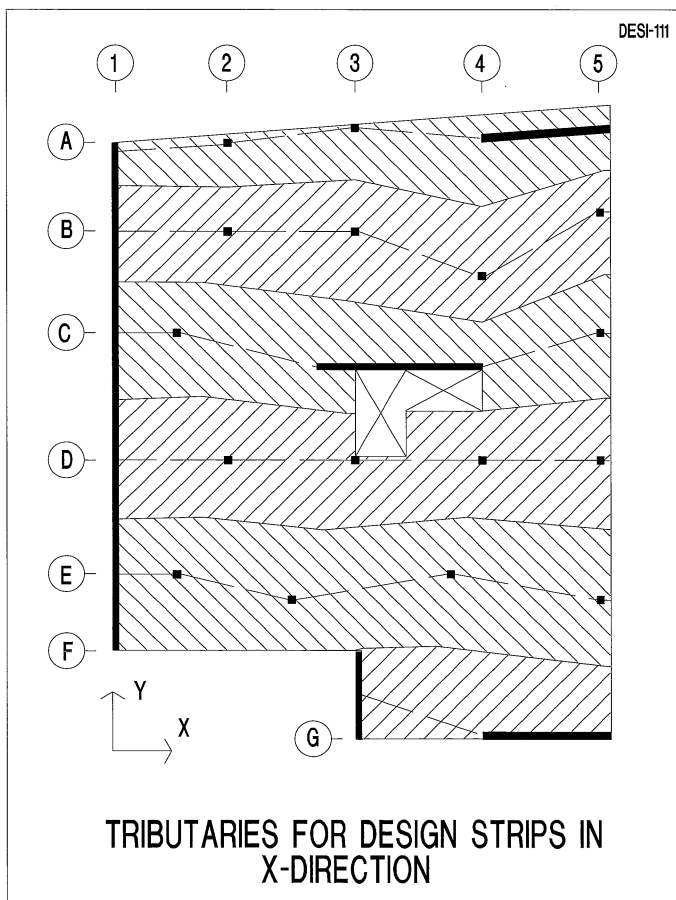


FIGURE 2.2-5

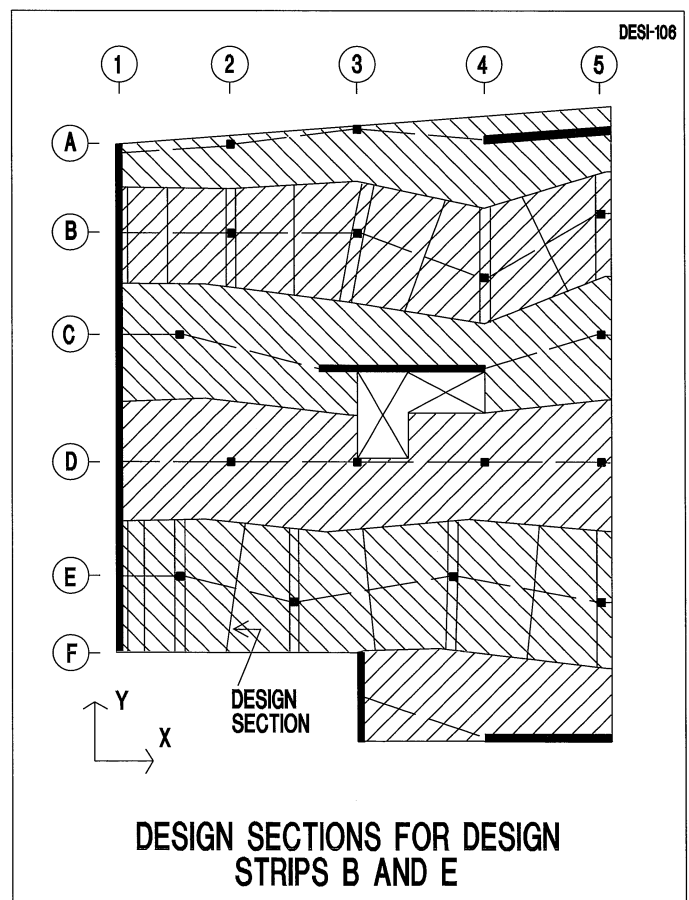


FIGURE 2.2-7

supports, sections can be chosen at the face of support to take advantage of the reduced actions away from the support centerlines.

2.3 Analysis Options

In the analysis step, the actions are distributed among the load paths (design strips) to satisfy the equilibrium of the applied loading.

2.3.1 Frame Analysis

In both the Simple and Equivalent frame options, each design strip is extracted from the floor and re-constructed with appropriate support conditions and loading to create an approximated frame model. Each design strip is analyzed as an independent structural system, isolated from the adjacent design strips.

Consider design strip B, shown as a separate entity in plan (Fig. 2.3.1-1) and in elevation (Fig. 2.3.1-2). For plane frame analysis, the strip is straightened along its line of support as illustrated in Fig. 2.3.1-1(b). The span length thus corresponds to the slant distance between adjacent supports. Note that the tributary widths may vary over a single span. To simplify the analysis, these varying tributary lines are typically idealized as straight boundaries (Fig. 2.3.1-1(c)). Usually, the idealized tributary is chosen to be

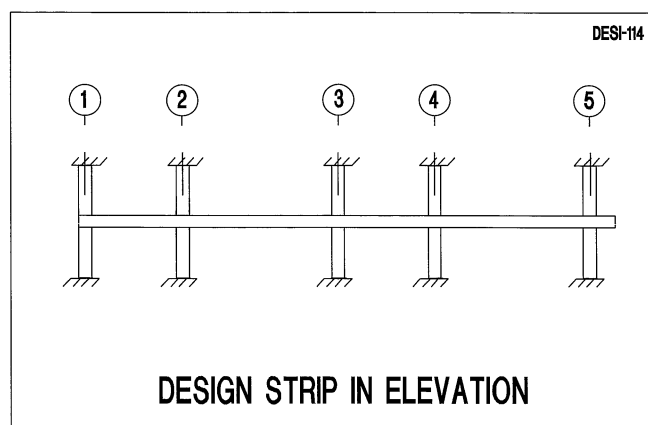


FIGURE 2.3.1-2

conservatively larger than the actual tributary. If the change in tributary width in any given span varies by more than 20%, it may be worthwhile to model the tributary as a series of steps to reduce the reinforcement required. Additional approximations may be necessary for other non-standard conditions.

2.3.2 Finite Element Analysis

As noted above, if either the Simple Frame or the EFM is used, each design strip must be extracted from the floor system and analyzed as a plane frame. With the FEM, the entire floor can be analyzed at one time. The results of an FEM analysis must be processed as "design strips" and "design sections" for code stipulated serviceability and strength checks, however. As with the frame methods, the design strips are based on the assumed load paths. The design strips do not need to be selected before the analysis though. This can sometimes be advantageous since the results of the FEM analysis can be used to select design strips which are more in line with the natural (assumed elastic) response of the slab.

Ideally, a design strip should be bounded by lines of zero shear transfer since this ensures that each design strip is designed to carry only the loading that is located directly on it. For the floor slab used in this example, the flow of the loading to the supports is shown by arrows in Fig. 2.3.2-1. The arrows are normal to the planes of maximum vertical shear; the length of each arrow indicates the magnitude of the shear. Figure 2.3.2-2 shows the lines of zero vertical shear transfer in the Y-direction as determined using the flow of loading shown in Fig 2.3.2-1. The alternate hatched and clear regions indicate the natural load tributaries for the support lines A through G.

Displays such as the one in Fig. 2.3.2-2 that show the "natural tributaries" allow the calculated actions to be assigned to the design strips in accordance with the elastic response of the structure. A design based on these tributaries is likely to be more economical with respect to material usage, especially if the floor configuration

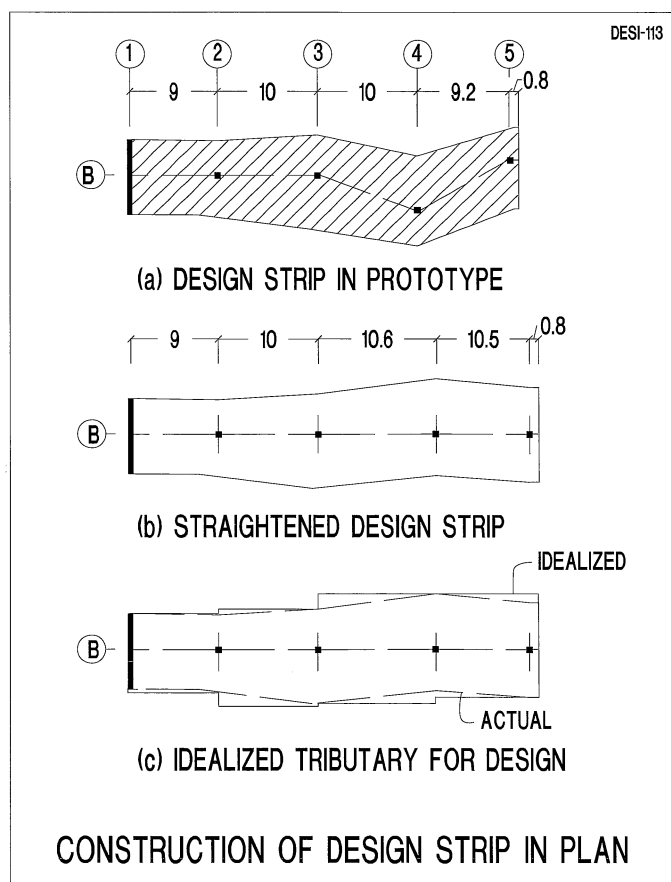


FIGURE 2.3.1-1

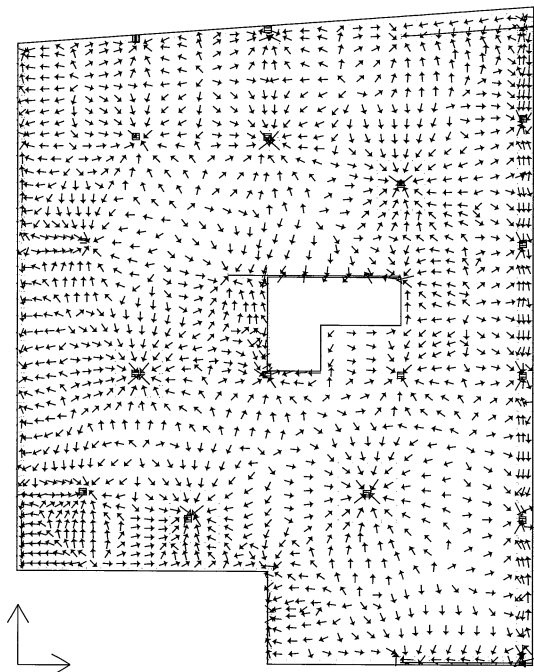
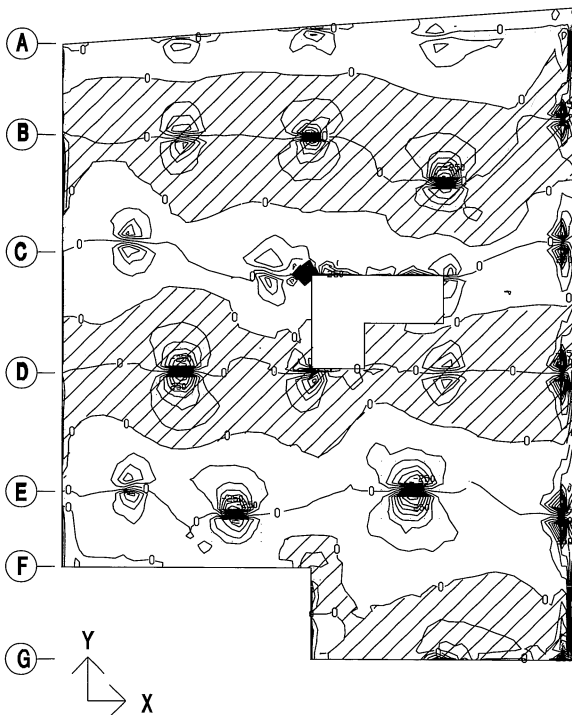


DIAGRAM OF LOAD FLOW UNDER SERVICE CONDITION

FIGURE 2.3.2-1



**ZERO LINE OF SHEAR TRANSFER
IN Y-DIRECTION**

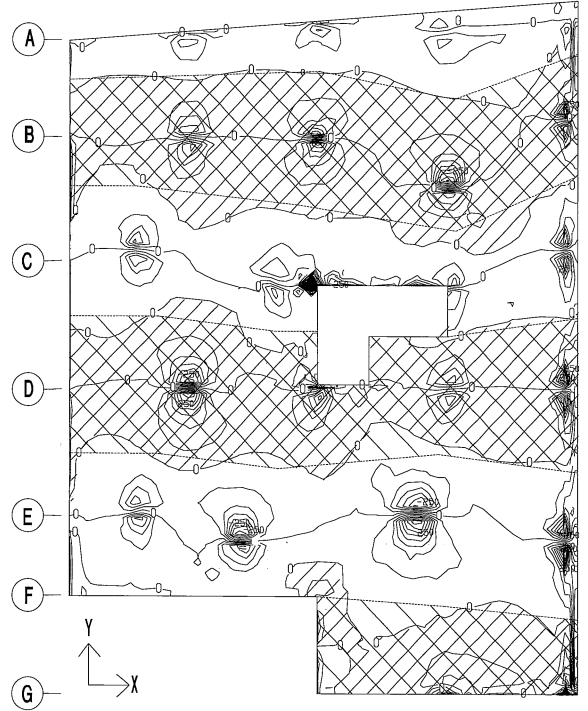
FIGURE 2.3.2-2

is irregular. Design strips are typically based on the standard support lines and tributaries described before however. In most cases, the increased design effort required to select natural tributaries outweighs the benefits of the refined design strips. Figure 2.3.2-3 shows the design strips selected by the procedure outlined for the frame methods superimposed on the "natural tributary" lines of the floor.

2.4 FEM Versus Frame Methods

The design procedure for a FEM analysis is very similar to that of the frame methods. It consists of selecting load paths leading to design strips and design sections. This is followed by the determination of demand actions for each design section. The design step consists of application of the demand actions to the respective design sections.

As with the frame methods, the objective of an FEM analysis is to provide information for a safe and serviceable design in accordance with the prevailing code(s). The amount of information obtained from a FEM analysis is generally more than that required by code for serviceability and safety checks, however. In particular, the FEM provides more accurate information on the floor system's response to applied loading.



**ASSUMED DESIGN STRIPS SUPERIMPOSED
ON NATURAL TRIBUTARIES IN X-DIRECTION**

FIGURE 2.3.2-3

2.4.1 Determination of Design Moment

To perform the strength evaluation of a design section, the design moment at the section is applied to the entire cross-sectional area of the section. With the FEM, design moments are determined by integrating the moment distributions across the design section. As an example, refer to Fig. 2.4-1 which is an enlargement of a corner of the floor slab example. Observe the design strip B, and the variation of moment M_y along the three design sections, one at the face of each support and one in the field of span 1-2. The moment used for the determination of reinforcement and stresses at each design section is the area (integral) of the moment distributions shown. For example, at the face of support at line 2, the design moment is $M_y = 281 \text{ kNm}$.

2.5 Structural Detailing

The analysis and design steps discussed above determine the area of primary reinforcement required for each design strip. To ensure satisfactory performance under service conditions, it is essential that this reinforcement be properly distributed. Distribution of the primary reinforcement is considered to be Structural Detailing.

2.5.1 Nonprestressed Floors

The ACI [ACI-318, 1999] recommendations for reinforcement layout are based on the column-strip, middle-strip concept.

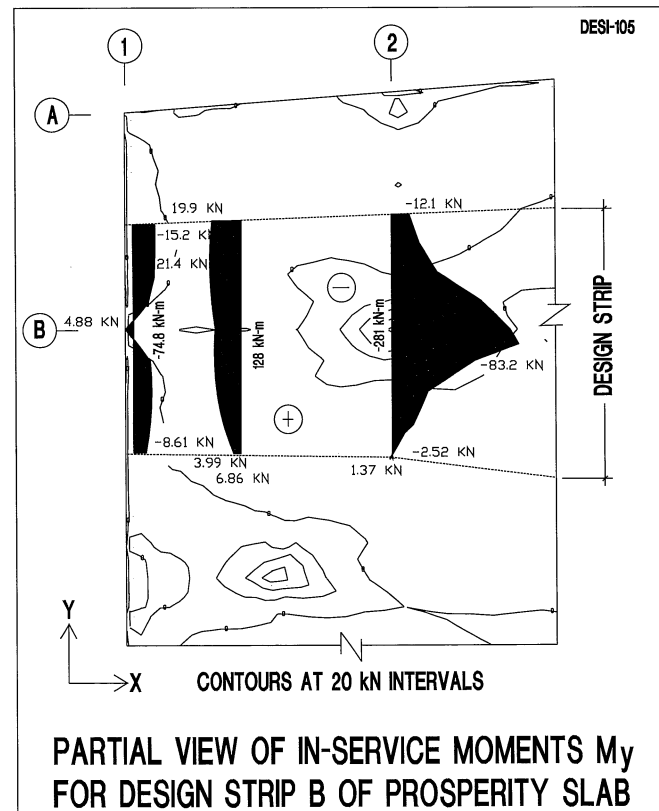


FIGURE 2.4-1

Although these recommendations are valuable guidelines, they cannot always be applied when the column plan is irregular or there are discontinuities such as large openings. The following are some of the underlying principles for determining reinforcement layout; they can be used as guidelines when a strict column-strip, middle-strip distribution will not work.

- Distribute the required reinforcement as close as practical to the distribution of the moment. It is not critical to follow strict percentages as long as the required area of reinforcement is accounted for. For the design strip shown in Fig. 2.4-1, for example, the reinforcement at the face of column at line 2 should be concentrated in the column region, while in the field, the distribution should be essentially uniform.
- At the exterior columns of a column-supported slab, place the entire amount of reinforcement within the column region. Place the majority of the reinforcement within the column region at interior columns. The width of the column region should be based on engineering judgment; one half of the tributary width is usually a reasonable value.
- The reinforcement everywhere along a design strip must be at least equal in both area and spacing to that required by code for shrinkage and temperature.

2.5.2 Post-Tensioned Floors

The layout of the post-tensioning and mild steel reinforcement within a design section is more flexible for post-tensioned floors than for nonprestressed floors. A detailed account of reinforcement layout for post-tensioned floor systems is given in [Aalami, 2000]. The principal guidelines for two-way post-tensioned slabs are as follows; for additional details refer to the code [ACI-318, 1999].

- Place the top mild steel reinforcement for both directions in bands over the columns. These bands are typically narrower than the column regions used for nonprestressed floors.
- Place bottom mild steel reinforcement where convenient for construction.
- Place the post-tensioning tendons where convenient, provided there are two tendons over the columns in both directions and the tendon spacing in one direction (the "uniform" direction) is not greater than eight times the slab thickness.
- Where average compression is likely to be less than a minimum value (100 psi, approx 0.7 MPa), provide mild steel or a combination of prestressing and mild steel to control shrinkage and temperature cracking.

2.5.3 Determination of Additional Mild Steel Reinforcing (Passive Reinforcement)

Structural Detailing also includes determination of the additional reinforcing steel required for both crack control and development of the load paths. Some of the additional steel, i.e. trim steel around small openings, can be covered by typical details on the structural drawings. In general, however, the additional steel must be determined by an engineer who has a good understanding of how the slab will respond to all likely load conditions. Both experience and sound engineering judgment are required.

Additional steel is typically required over any support that is not part of a designated load path. In addition, "distribution" steel perpendicular to the primary reinforcement is typically required under concentrated loads. In post-tensioned floors, additional mild steel may be required in areas where restrictions on access or unusual geometries make it difficult to provide sufficient post-tensioning.

2.6 Waffle Slabs

Post-tensioned waffle slabs (Fig 2.6-1) provide added economy where waffle forms are readily available and concrete is relatively expensive. Waffle slabs are designed using the concept of load path designation discussed above. If a frame analysis method is used, each design strip is represented by an idealized geometry as

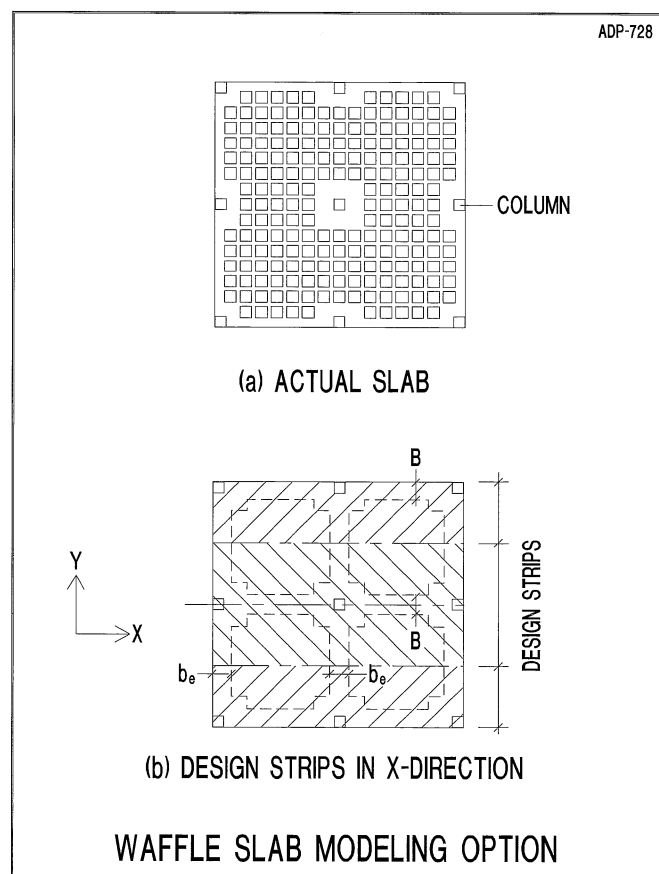


FIGURE 2.6-1

shown in Fig 2.6-1(b). The idealized geometry has the same area and moment of inertia as the actual structure in the direction of the frame; it has approximately the same torsional stiffness in the perpendicular direction.

When using Finite Element analysis there are two options. The floor system can be modeled faithfully with each waffle represented by its true geometry. Alternatively, the waffle stems can be lumped together and positioned along the lines of supports to represent the same area, moment of inertia and section moduli as the actual structure (Fig. 2.6-2). When the second option is used, the prestressing tendons are banded in the lumped stems along the support lines for design purposes.

Tendon layout in waffle slab construction follows the same general procedure as flat slabs. The preferred procedure is to place a minimum of one tendon in each waffle stem in one direction. In the perpendicular direction, the tendons are banded along the support lines. Unlike the typical tendon layout for flat slabs, uniform distribution of the tendons among the waffle stems in both directions is also widely used.

The dimensions of the structural model for a waffle slab are given in Appendix A

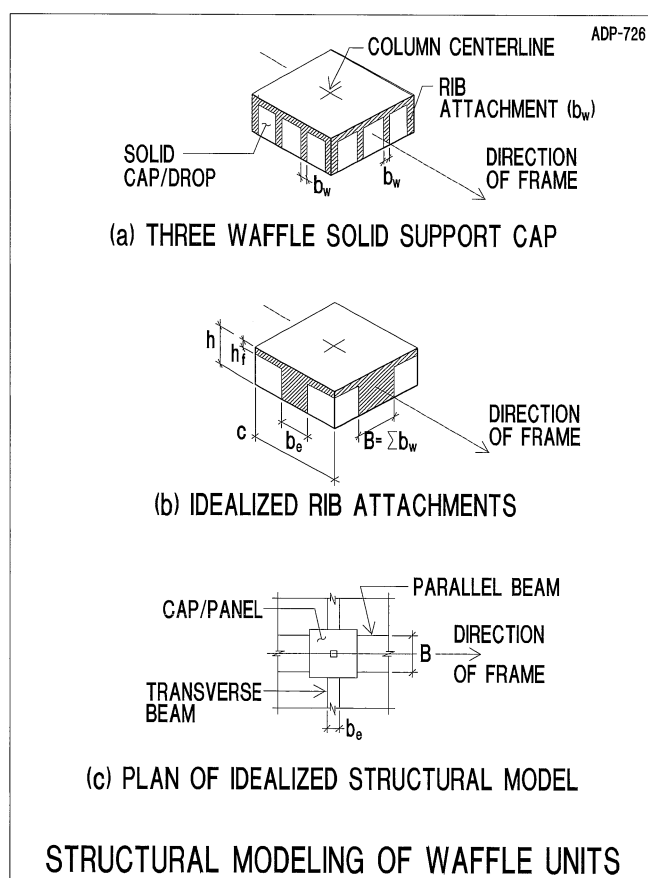


FIGURE 2.6-2

2.7 Summary

The key features of the concrete floor design process are as follows:

- There are three commonly used analysis methods: the Simple Frame, the Equivalent Frame, and the Finite Element methods. The Equivalent Frame method is a refinement of the Simple Frame method which reduces the column stiffness to account for biaxial plate bending.
- Regardless of the analysis method used, the floor system must go through a structural modeling step, in order to designate "Design Load Paths". Load paths must be designated before doing a Simple or Equivalent Frame analysis, but can be designated either before or after a Finite Element analysis.
- Design strips are selected in accordance with the designated load paths. The outcome of the analysis step is the moments and shears that must be resisted by each design strip.
- At the design stage, the entire (integral) moments at a design section is applied to the design section in order to calculate the reinforcement required.
- Calculation of the reinforcement is followed by "Structural Detailing," which ensures the serviceability of the floor and the implementation of the design concept.

All three analysis methods are variations of the proven "strip method," which requires choosing a statically admissible stress field that satisfies equilibrium. Thus, all three yield a lower bound (safe) solution. Recognition of common features in the three design methods, in particular the necessity of selecting "load paths", is essential to the automation of concrete floor design.

3. CONCLUSION

The characteristic features of concrete design were discussed and compared with design of other materials. It was concluded that the selection of a load path is a prerequisite for the design of a concrete floor. Using the proven concepts of the strip method, it was shown that the selection of the load paths, and hence the floor design could be automated.

The automated part of the design needs to be followed by Structural Detailing, however. Structural Detailing involves adjustments in the position of calculated reinforcement and placement of any additional reinforcement required to carry out the design concept. It is thus highly dependent on engineering judgment. Guidelines for Structural Detailing of both prestressed and nonprestressed floors were given.

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APPENDIX A

The cross-section of a waffle slab is shown in Fig. A-1. For design, the waffle slab is idealized as illustrated in Figs. 2.6-1(b) and 2.6-2. The width of the beam in the direction of the analysis, B , is the sum of the waffle webs and any solid regions. In the transverse direction, a torsional member is created by lumping all of the stems joining the solid column block into a rectangle with effective width, b_e .

The column support region is modeled as show in Fig. 2.6-2(b) with a plan view in Fig. 2.6-2(a). The effective width, b_e , is determined so as to provide a torsional stiffness equal to the sum of the individual stiffnesses of the stems meeting the solid block. The effective width, b_e , of the torsional member is determined using the assumption that the equivalent torsional member consists of a relatively thin flange (c , h_f) and a thin stem (b_e , $h-h_f$). The torsional constant of each of the constituent rectangles is one-third of its lengths times its thickness cubed.

The torsional stiffness, J , of the actual slab is given by:

$$J = (ch_f^3/3) + \sum[h-h_f]b_w^3/3 \quad (1)$$

Depending on the aspect ratio of the structural model, one of two relationships is used for the calculation of b_e , the width of the equivalent torsional stiffness member.

(i) If $b_e < (h-h_f)$

$$J_e = (ch_f^3/3) + b_e^3(h-h_f)/3 \quad (2)$$

Setting J_e equal to J from Eqn (1) and solving for b_e gives:

$$b_e = (\sum b_w^3)^{1/3} \quad (3)$$

(ii) If $b_e > (h-h_f)$

$$J_e = (ch_f^3/3) + b_e(h-h_f)^3/3 \quad (4)$$

Setting J_e equal to J from Eqn (1) and solving for b_e gives:

$$b_e = \sum b_w^3 / (h-h_f)^2 \quad (5)$$

Dimensions are as shown in Fig. A-1.

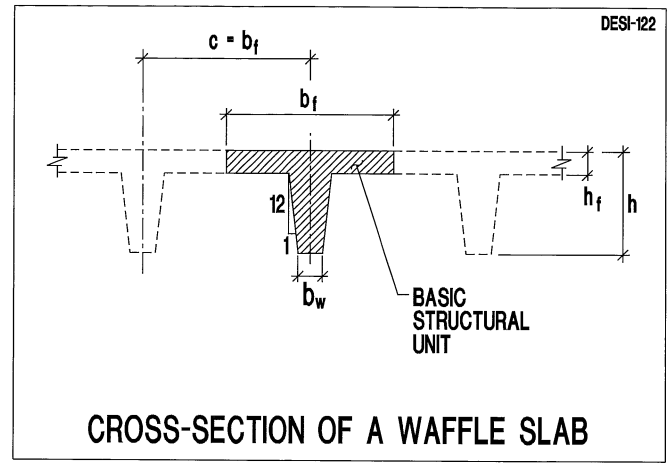


FIGURE A-1



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