Key Aspects of 2D and 3D Finite Element Solutions for Serviceability and Strength Requirements of Post Tensioned Beams and Slabs

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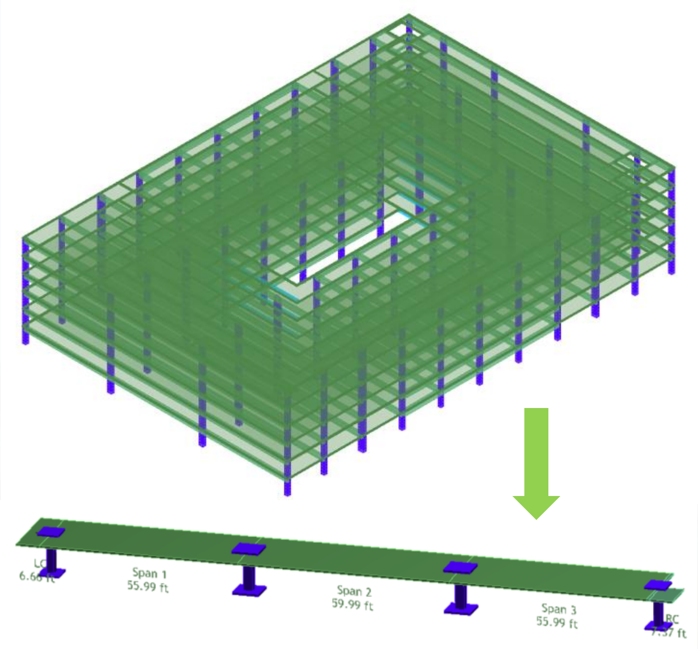
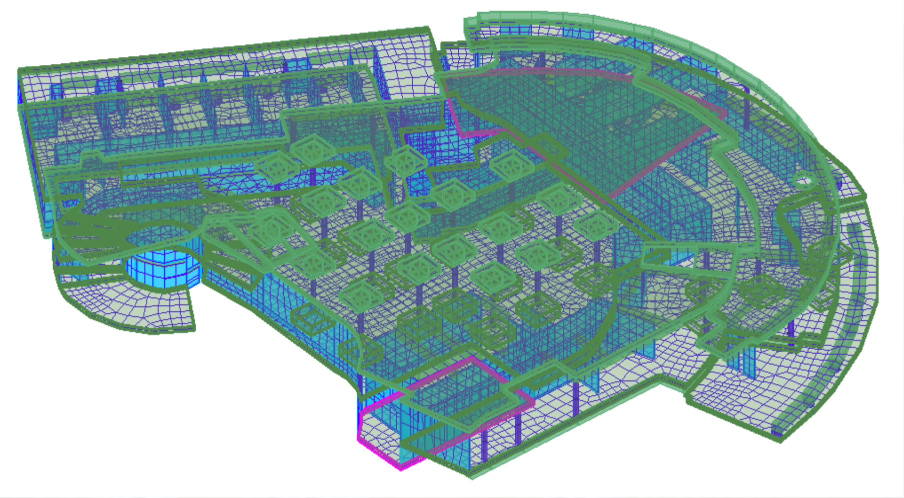
Post-Tensioning Project Variance

The design of two-way unbonded or bonded post-tensioned two-way flat slab and flat plates, one-way slabs and beams oftentimes relies on computer models and analysis for checking compliance of code-specific serviceability requirements and strength design.

Two common software approaches are the use of a simplified 2D frame program employing the Equivalent Frame Method or an advanced 3D Finite Element program for continuum slabs with and without integrated beams.





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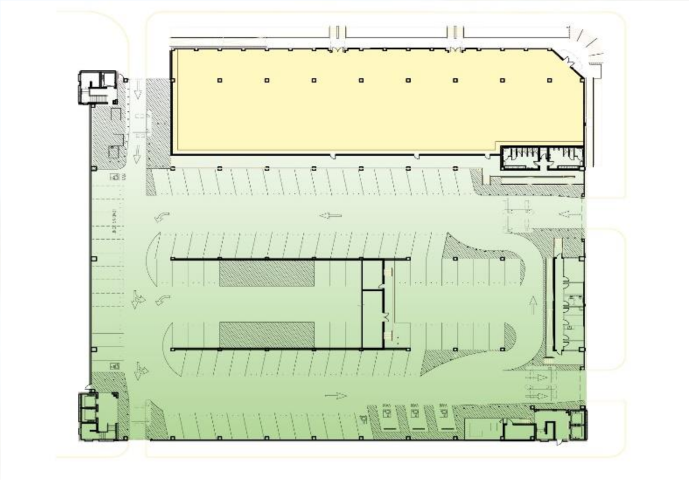
Structural Geometry – 2D Frame Approach

Key Attributes:

- Linear and repetitive layout of beams, columns, slabs
- Redundant geometry level-to-level
- Flat surfaces, sloped or not
- Regular loading patterns – uniform loads, skipped/non-skipped
- Easily defined load path and generation of design strips
- Beam and one-way slab systems
- Conducive to methodology of Equivalent Frame Method

2D Frame Approach – Parking Structures



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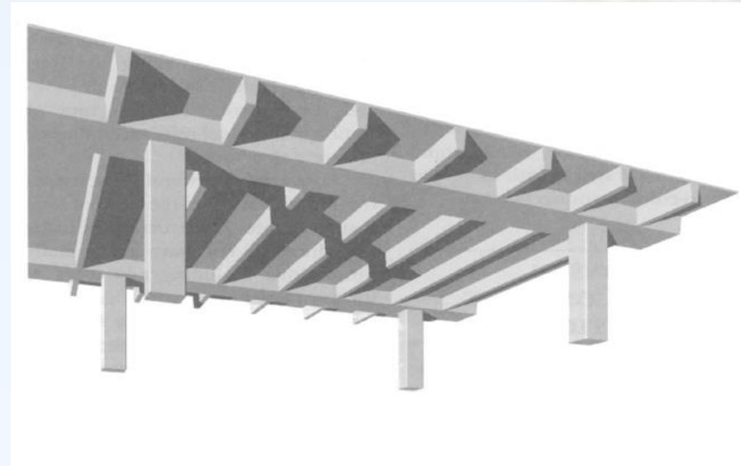
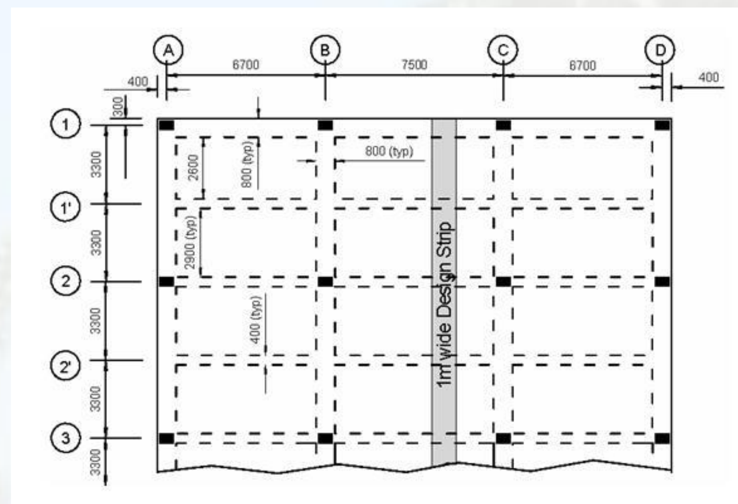
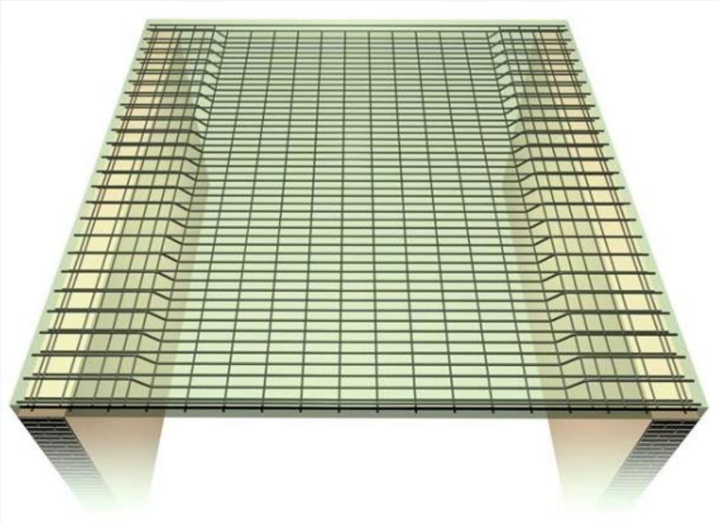
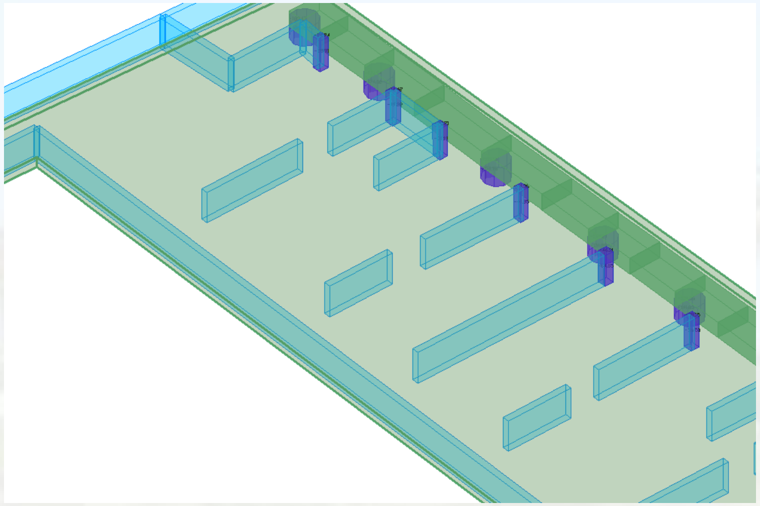
2D Frame Approach – One-Way Slab and Beam





2D Frame Approach – Wall-Supported One-Way Slabs





Structural Geometry – 3D Finite Element Approach

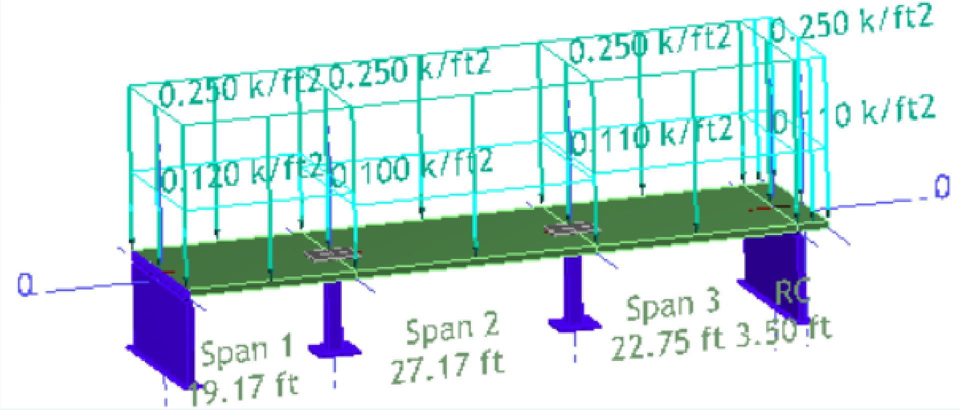
Key Attributes:

- Supports any geometry and component configuration
- Ideal for un-symmetric and geometrically complicated two-way slab systems
- Folded slabs, slab depressions, changes in thickness
- Directly accounts for concentrated loads in mid-panel locations
- Irregular support arrangements
- Irregular loading patterns
- Flexibility in load path and strip generation
- Accuracy in deflection behavior and prediction

3D Finite Element Approach– Two-Way Flat Plates and Slabs

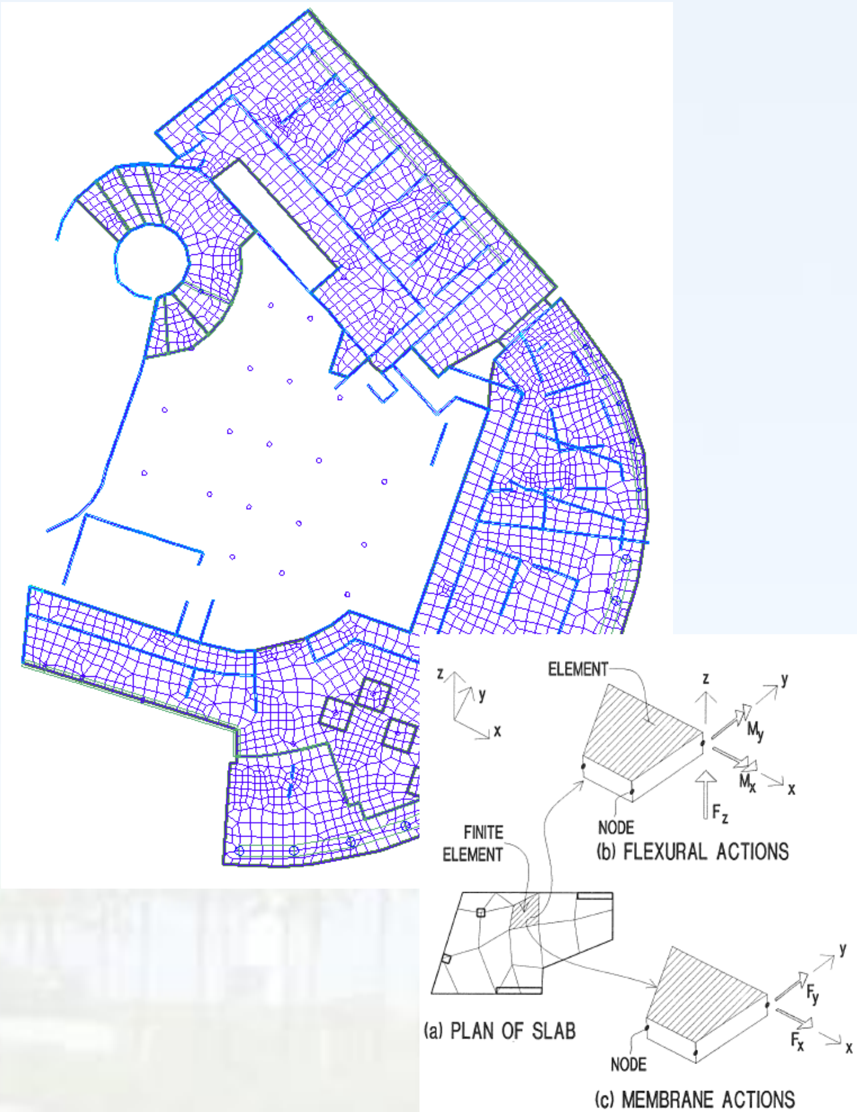
3D Finite Element Approach– Two-Way Mat Foundations

Analysis Methodology– 2D Frame Approach



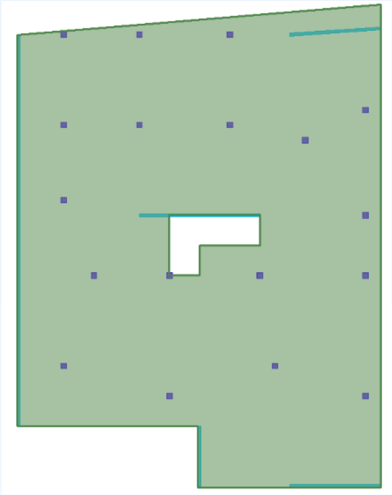
- Simplified analysis approximations – EFM and other simulated joint conditions, non-prismatic frame elements, equivalent column
- Loading discretized along frame reference line
- Limited stiffness formulation – bending and shear
- Does not capture out-of-plane bending and in-plane axial stiffness – no membrane action
- Requires defining and modeling of individual frames in each direction
- Post tensioning modeled as equivalent balanced loads applied as external loads
- Deflections are dependent on inherent formulation – don’t account mid-panel rotations and displacements transverse to frame
- Punching shear – checks bending only about 1 local support axis and requires enveloping of multiple frame lines.

Analysis Methodology– 3D FEM Approach

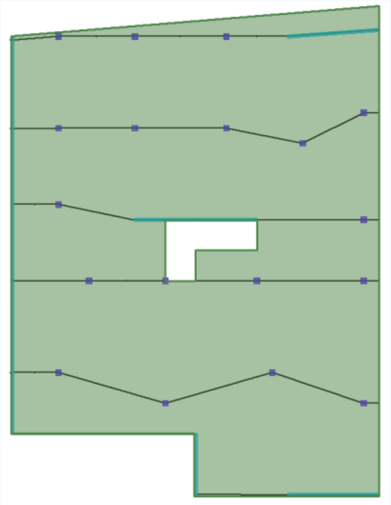


- Elements (shells) and nodes account for flexural (Mx, My, Fz) and axial (Fx, Fy)
- Loading can be modeled at any location and in multiple forms and directions – line, point, uniform
- Loads are discretized to nodes – user controls density of elements for analytical refinement
- Stiffness of global matrix formulation accounts for multi-directional behavior
- Displacement-based element formulation
- User required to define support lines to determine tributary strips in X and Y
- Nodal integration used for calculation of section centroid forces
- Post tensioning modeled as internal element
- Deflections – linear elastic response or cracked solution both accounting for two-way slab behavior

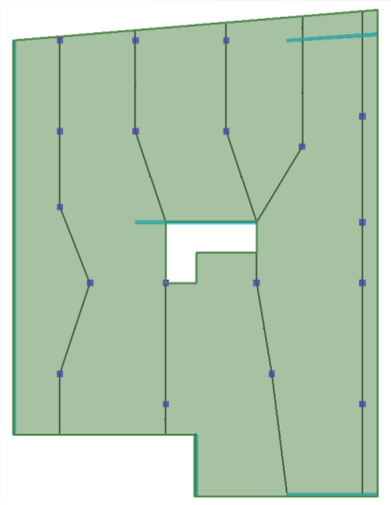
Generation of strip models for 2D and 3D approaches



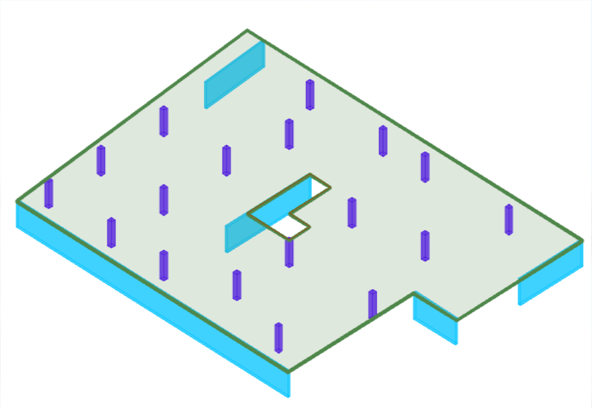
Two-way slab/plate

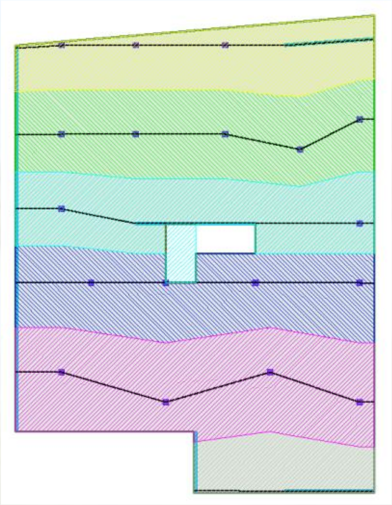


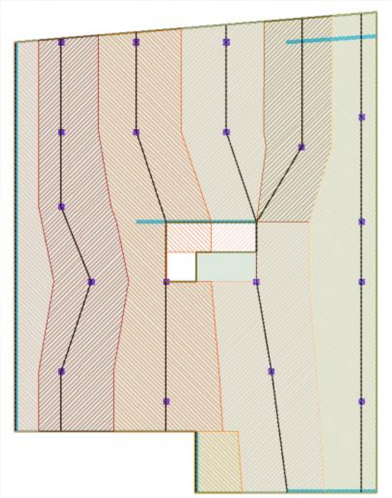
Support Lines – X
Tributary Region - Y



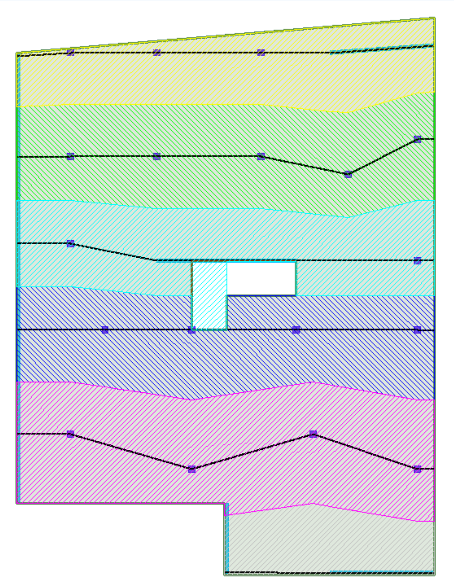
Support Lines – Y
Tributary Region - Y





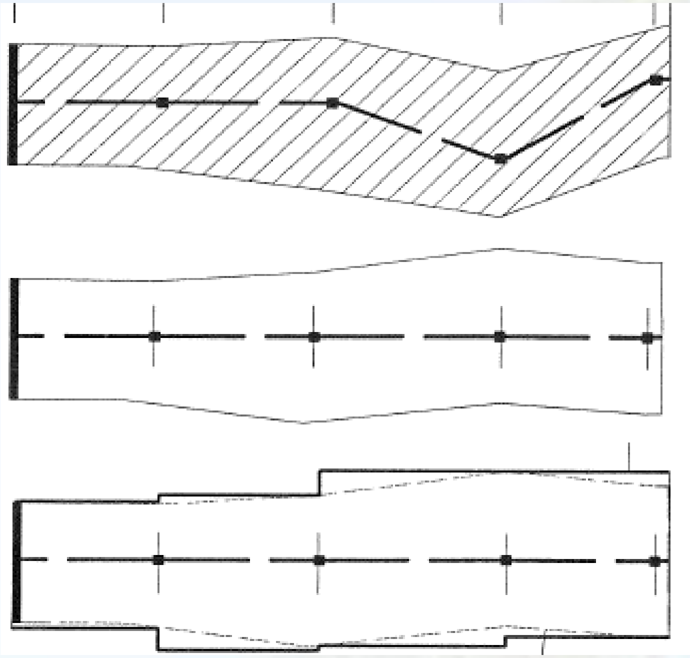


Idealization of strip models for 2D and 3D approaches

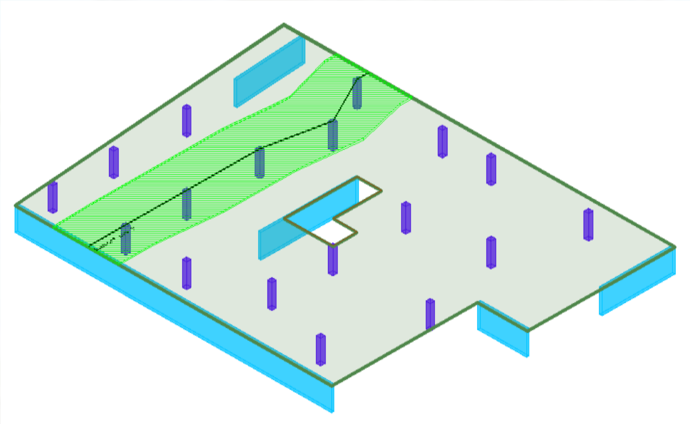


Generated Strips - X

Idealized strip in 2D environment requires approximation of each tributary span



Design strip in 3D environment requires no additional manipulation



Key Design Aspects
Serviceability and Strength requirements

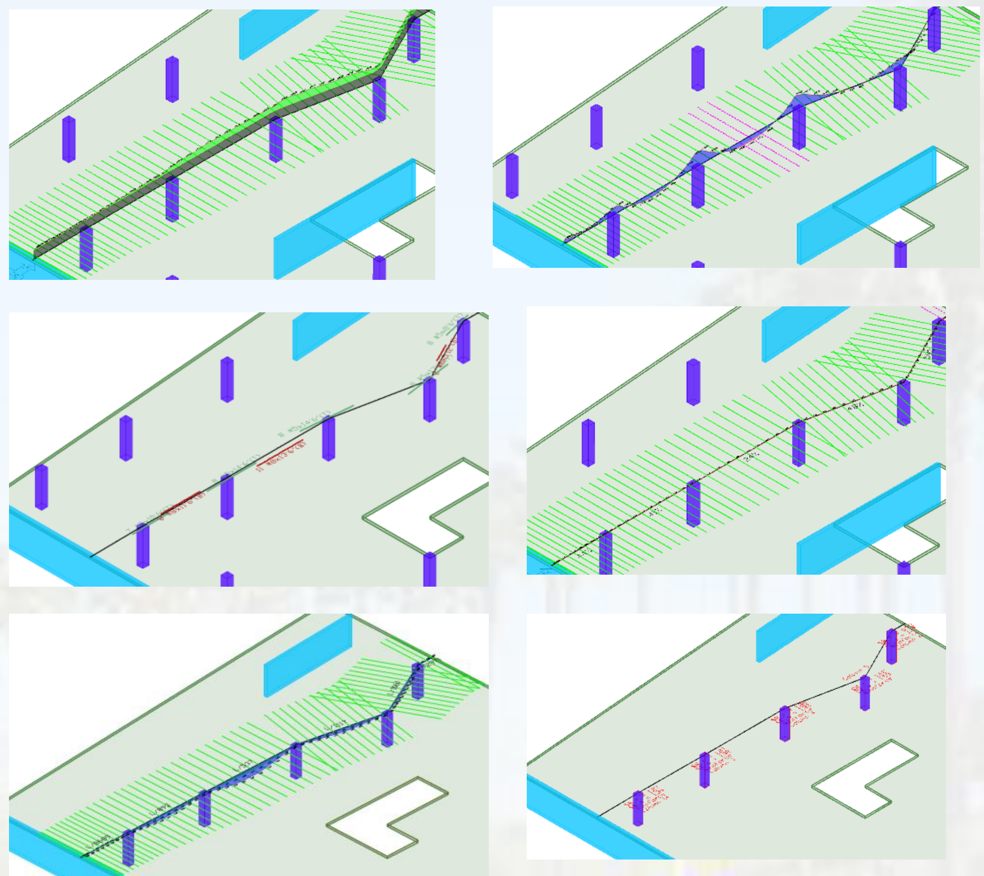
After design strips are determined and generation, the analysis and execution of the design is required for evaluation of serviceability and strength aspects for a slab and/or beams. While the design strips may be identical in geometry, material, properties, etc., the underlying analysis method used for determination of section forces may lead to differences in key design aspects shown below.

Serviceability

- Precompression (P/A)
- Extreme fiber stresses (Tension/Compression)
- Balanced loading
- Deflections
- Minimum reinforcement
- Initial stress at force transfer

Strength

- Ultimate reinforcement
- Punching shear



Serviceability - SLS

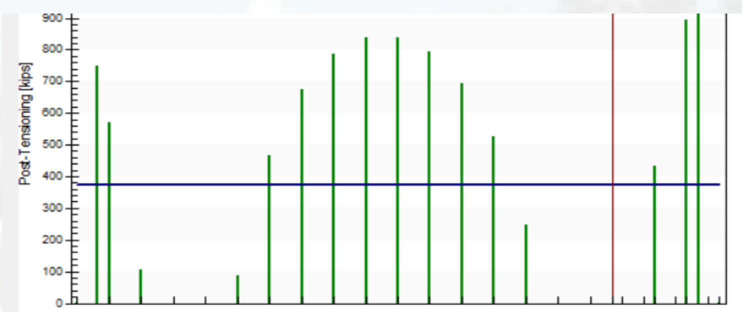
Precompression – P/A

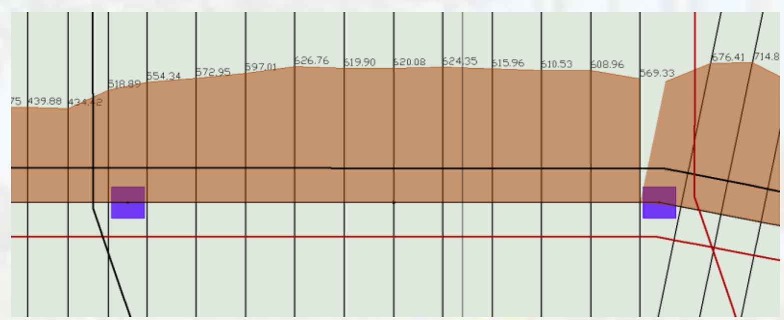
2D Frame Method

- Effective force or calculated force method
- Dependent on idealized geometry of modeled tributary – not necessarily “effective” tributary
- Uses calculated tendon force – does not account for restraint and loss of tendon force
- Independent of service load combination – hard-coded to use PT force only
- Inherently accounts for only tendons in the frame direction
- Assumes force deposited fully at each design section

3D Finite Element Method

- Effective force or calculated force method
- Dependent on actual geometry of the modeled design strip – not necessarily “effective” tributary
- Uses resultant centroid axial force of design section and accounts for loss of force due to restraint
- Dependent on service load combination – axial force used is that due to all load case parts that comprise the combination
- Centroid axial force due to primary tendons and other tendons in transverse direction
- Force distribution considered in elements



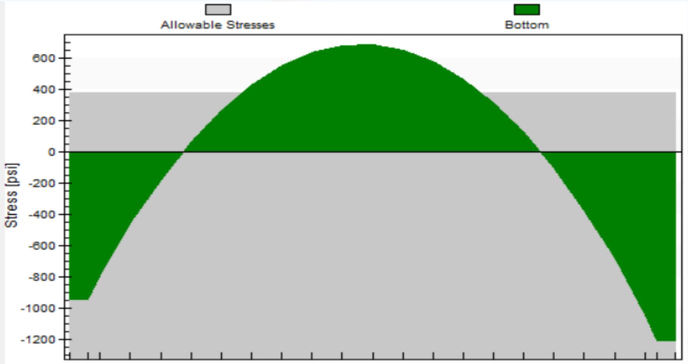


Serviceability - SLS

Allowable extreme fiber stresses – Tension and Compress

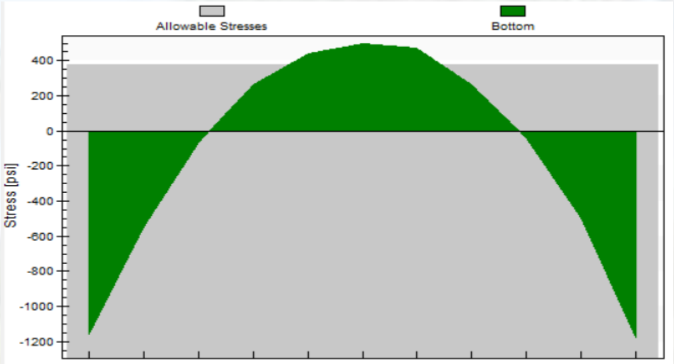
2D Frame Method

- Mp due to constant tendon force for primary action
- Frame analysis and resulting moments based on effective section properties – M/S ± P/A
- Section properties based on effective section properties
- Precompressive force is constant for centroid stress value
- Stress in reinforcement ignored
- Long-term effects not accounted for – no time dependency



3D Finite Element Method

- Mp due to varying tendon force for primary action
- Analysis and resulting actions based on FEM method and nodal integration over full design section - M/S ± P/A
- Section properties based on full section properties
- Precompressive force can vary for centroid stress value
- Stress in reinforcement ignored
- Long-term effects not accounted for – no time dependency



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Serviceability - SLS
% Balanced Dead Load

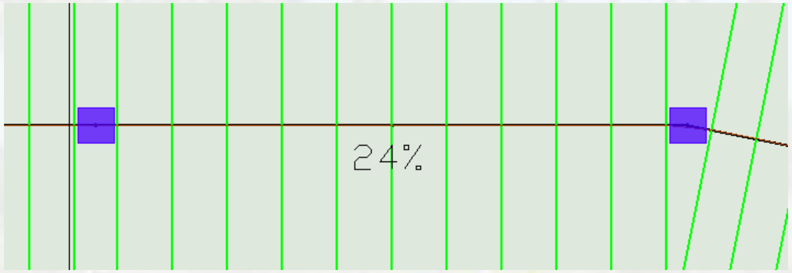
2D Frame Method

- Mp due to constant tendon force for primary action
- Frame analysis and resulting prestressing moments based on effective section properties
- PT balanced loads determined as external loading set due to force and profile of tendons
- Linear load from self-weight (SW) and dead load are discretized to frame centerline
- % reported may include contribution of both SW and superimposed dead load
- Does not account for tendons in transverse frame direction

	Number of strands	PT Force per unit width	PT Force	P/A	%DL balanced
CL	14	13.6	374.0	142	12
1	14	12.5	374.0	130	33
2	14	12.5	374.0	130	23
3	14	11.8	374.0	123	31
4	14	11.9	374.0	124	39

3D Finite Element Method

- Mp due to varying tendon force for primary action
- Analysis and resulting actions based on FEM method
- PT is considered and internal, resisting element with tendon segment discretization
- PT balanced loads are dependent on mesh density within a span tributary region
- Nodal forces are used in determining the total balanced load
- % reported may include contribution of both SW and superimposed dead load
- Accounts for tendon forces in transverse direction



Serviceability - SLS

Deflections – cracking and long-term effects

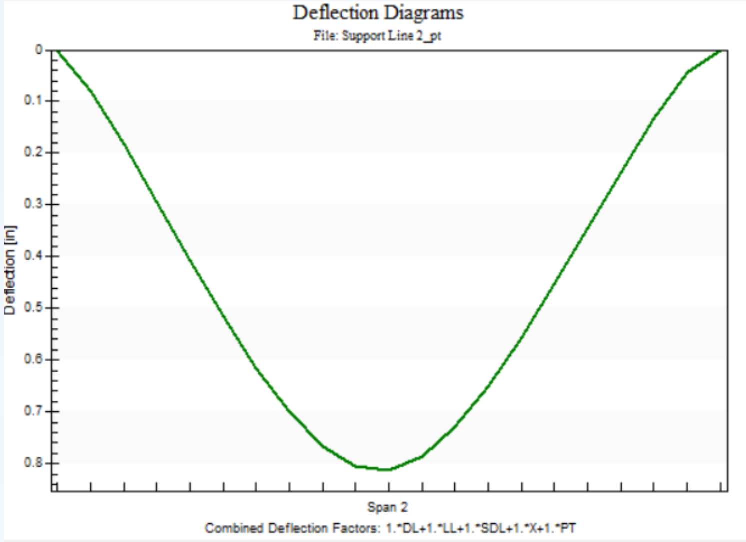
2D Frame Method

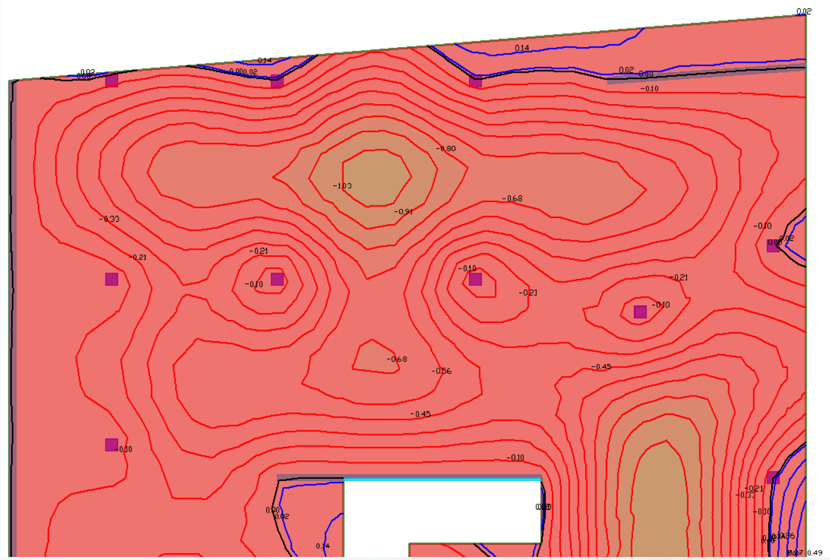
- Mp due to constant tendon force for primary action
- Frame analysis and resulting deflections based on effective section properties
- 2D frame considered by EFM – underlying methodology
- Consideration of cracking relative to direct frame results – Ma <> Ma → Ieff
- Considers reinforcement for Icr relative to single frame
- Long-term effects assume creep multipliers - no load history or sequencing
- No direct consideration of mid-panel displacements – loads discretized to frame centerline
- No multistory effects due to planted supports or immediate, elastic shortening of columns

3D Finite Element Method

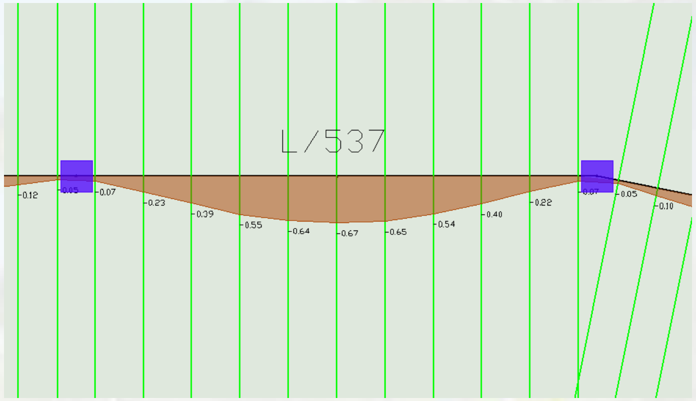
- Mp due to varying tendon force for primary action
- Analysis and resulting deflections based on FEM method
- 3D continuum by FEM considers bidirectional flexure and captures displacements at all mesh and frame element nodes – mid-panel
- Consideration of cracking relative to density and location of design sections in the “X” and “Y” directions and reliant on section results based on nodal integration
- Considers reinforcement for Icr relative to entire model
- Long-term effects may or may not use load history or sequencing. Creep factors may be applied
- Can consider multistory effects of planted supports and column shortening
- Generally allows for more accurate control and flexibility of deflection combinations to be considered

Serviceability - SLS
Deflections – cracking and long-term effects





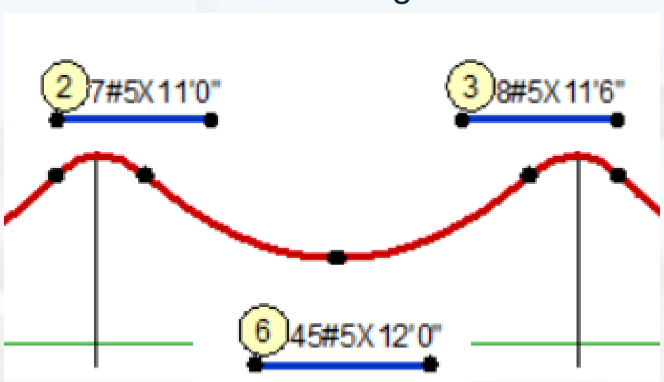
Span	Total
	in
CL	5.15(36)
1	-0.28(1061)
2	0.81(443)
3	0.09(3335)
4	0.17(1598)
CR	-0.11(542)



Serviceability - SLS
Minimum reinforcement

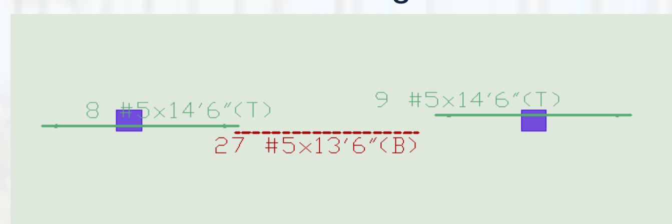
2D Frame Method

- Top reinforcement at supports based on full tributary area – not effective
- Bottom reinforcement is dependent on tensile stress
- Mp due to constant tendon force for primary action
- Frame analysis and resulting moments and stresses based on effective section properties
- Assumed reinforcement distribution over full tributary width
- Reinforcement follows global directions



3D Finite Element Method

- Consideration of reinforcement can be dependent on support line “support” nodes
- Top reinforcement based on full tributary for idealized tributary
- Bottom reinforcement is dependent on tensile stress
- Mp due to varying tendon force for primary action
- Analysis and resulting moments and stresses based on FEM method and nodal integration over full tributary
- Assumed reinforcement distribution over full tributary width
- Reinforcement follows design strip directions – can be converted to global



Serviceability - SLS
Initial force transfer condition

2D Frame Method

- Mp due to constant tendon force for primary action
- Frame analysis and resulting moments and stresses at initial condition based on effective section properties
- Calculating Nc - top and bottom reinforcement dependent on section stress zone in tension – effective section used
- Reinforcement follows global directions

3D Finite Element Method

- Mp due to varying tendon force for primary action
- Analysis and resulting moments and stresses at initial condition based on FEM method and nodal integration over full tributary
- Calculating Nc - top and bottom reinforcement dependent on section stress zone in tension – full section used
- Reinforcement follows design strip directions – can be converted to global

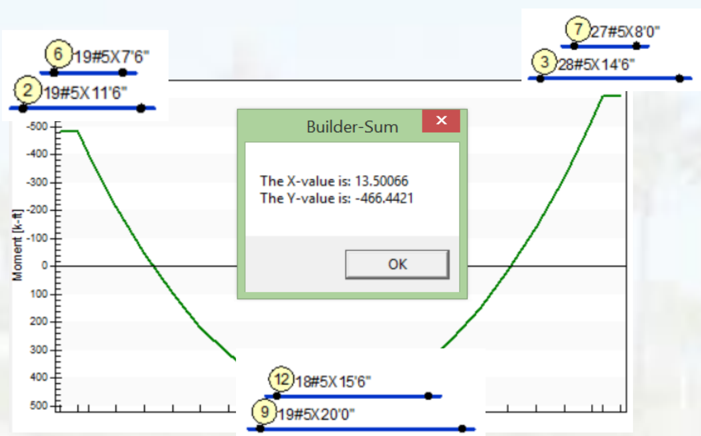
Strength - ULS
Ultimate reinforcement

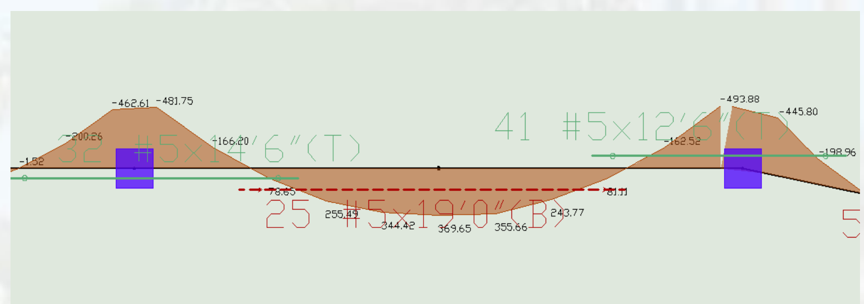
2D Frame Method

- Frame analysis actions based on effective section properties and EFM
- Net section tensile force not calculated or considered
- Secondary actions due to PT considered – dependent on constant force
- Assumed reinforcement distribution over full tributary width
- Reinforcement follows global directions
- Can have differing density of design sections

3D Finite Element Method

- Analysis and resulting actions based on FEM method and nodal integration over full tributary of design sections
- Net section tensile force calculated and considered for reinforcement calculation
- Secondary actions due to PT considered – dependent on varying force
- Assumed reinforcement distribution over full tributary width
- Reinforcement follows design strip directions – can be converted to global





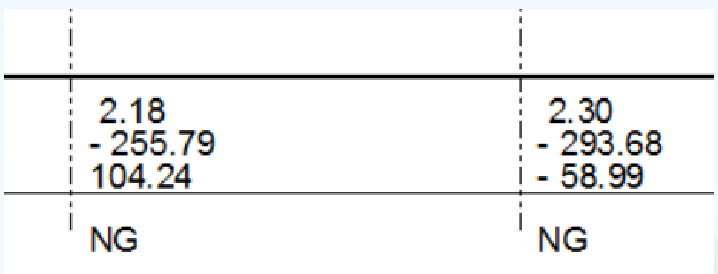
Strength - ULS
Punching Shear

2D Frame Method

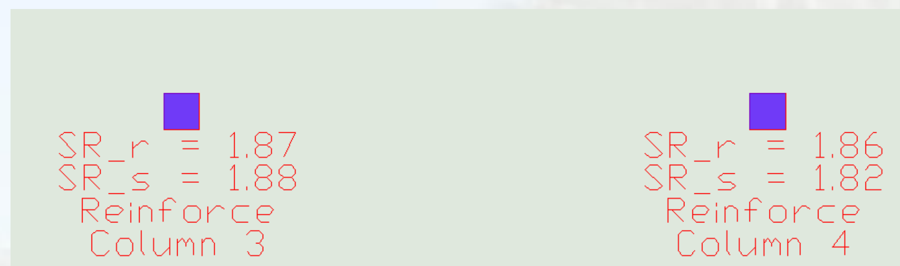
- Frame analysis actions based on effective section properties and EFM
- Out-of-plane moments not considered
- Enveloping of results required for multiple frame runs
- Secondary actions due to PT considered – dependent on constant force

3D Finite Element Method

- Analysis and resulting actions based on FEM method
- Direct FEM results used, not integrated section results
- Separate or combined stress from moment considered
- Secondary actions due to PT considered – dependent on varying force



Label	Layer	Cond.	Factored shear	Factored moment	Total stress	Stress ratio
			k	k-ft	ksi	
2	1	1	-255.79	+104.24	0.426	2.180
3	1	1	-293.68	-58.99	0.447	2.297



Label	Condition	Axis	Factored shear	Factored moment	Total stress	Stress ratio
			k	k-ft	ksi	
Column 3	Interior	rr	-280.585	-34.818	0.412	1.87
Column 3	Interior	ss	-280.585	35.868	0.413	1.88
Column 4	Interior	rr	-272.200	-47.774	0.409	1.86
Column 4	Interior	ss	-272.200	-35.077	0.400	1.82

Conclusions and Recommendations

- Both methods (2D Frame and 3D FEM) can be applied to different design types and configurations
- Selecting the most applicable design approach can lead to more accuracy in results – e.g. complex loading and geometry lends itself to a FEM approach
- Understand the programmatic and underlying assumptions of each analytical and design method implemented in software
- While the FEM method has gained wide acceptance and use, many engineers continue to employ simplified frame software
- Use of both methods can provide deeper insight into behavior of a system design
- In comparing results between both methods, it is critical to ensure that the input is identical – e.g. LLR, tributary width, support conditions, material properties, etc.
- There is no substitute for validation through other methods (hand-checks, spreadsheets, published tables, etc.)