APPENDIX A.8  
Design Example: Residential Slab on Compressible Soil

GIVEN: A single-story residence in Alexandria, Louisiana, with the dimensions as shown. Construction is wood frame with concrete masonry units for exterior walls and sheetrock interior, with foundations built on polyethylene sheathing.

E. Design Slab (1) only (for illustrative purposes). Assume spacing of stiffening beams as shown below.

A.8.2 Edge Lift Design (Predominant Distortion Mode)  
A. Approximate Depth of Stiffening Beam

Note: Experience has shown that an initial assumption of \( e_m = 1 \) will yield a satisfactory trial section.

Assume:
\[ e_m = 1.0 \text{ ft. (for initial estimate of beam depth only)} \]
\[ y_m = 0.75 \text{ inch (conservative since } \delta = 0.75 \text{ in.)} \]
\[ \beta = 4.0 \text{ ft.} \]

1. Long Direction
\[ C_A = 1,920 \text{ (Table 6.2)} \]
\[ L = 40 \text{ ft. } > 6\beta = 6(4) = 24 \text{ ft. USE 24 ft.} \]
\[ \Delta_{allow} = \frac{12(L \text{ or } 6\beta)}{C_A} = \frac{12 \times 24}{1,920} = 0.15 \text{ in.} \]
\[ h = \left( \frac{L^{0.35} S^{0.88} e_m^{0.74} y_m^{0.76}}{15.9 \Delta_{allow}^{0.04}} \right)^{1.176} \]
\[ h = \left( \frac{(40)^{0.35}(12.67)^{0.88}(1.0)^{0.74}(0.75)^{0.76}}{15.9 \times 0.15 \times 840^{0.04}} \right)^{1.176} = 16.2 \text{ in.} \]

2. Short Direction
\[ L = 38 \text{ ft. } > 6\beta = 6(4) = 24 \text{ ft. USE 24 ft.} \]
\[ \Delta_{allow} = \frac{12 \times 24}{1,920} = 0.15 \text{ in.} \]
\[ h = \left( \frac{L^{0.35} S^{0.88} e_m^{0.74} y_m^{0.76}}{15.9 \Delta_{allow}^{0.04}} \right)^{1.176} \]
\[ h = \left( \frac{39^{0.35} \cdot 13.33^{0.88} \cdot 10^{0.74} \cdot 0.75^{0.76}}{15.9 \times 0.15 \times 840^{0.01}} \right)^{1.176} = 16.8 \text{ in.} \]

Several iterations have been skipped for the sake of brevity.

Try \( h = 24 \) in.
\( b = 10 \) in.

**B. Check Soil Bearing**

1. **Allowable Soil Pressure:**
   \[ q_{allow} = 1,500 \text{ psf} \]

2. **Applied Loading:**
   - Slab weight = 40x38x0.333x150 = 76,000 lb.
   - Added DL = 15x40x38 = 22,800 lb.
   - Beam weight = 20x10x298.67/144x150 = 62,223 lb.
   - Perimeter Load P = 156x840 = 131,040 lb.
   - Live Load = 40x36x40 = 60,800 lb.
   - **Total** = 352,863 lb.

3. **Beam Bearing Area** = 298.67 ft x 0.833
   \[ = 248.8 \text{ sq. ft.} \]

4. **Soil Pressure** = 352,863/248.8
   \[ = 1,418 \text{ psf} < 1,500 \text{ psf} \text{ OK} \]

**C. Section Properties**

<table>
<thead>
<tr>
<th>Long Direction</th>
<th>Short Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam depth ( h ) (in.)</td>
<td>24</td>
</tr>
<tr>
<td>Beam Width (each) (in.)</td>
<td>10</td>
</tr>
<tr>
<td>Number of beams</td>
<td>4</td>
</tr>
<tr>
<td>Total Beam Width (in.)</td>
<td>40</td>
</tr>
<tr>
<td>Slab thickness (in.)</td>
<td>4</td>
</tr>
</tbody>
</table>

**Long Direction**

<table>
<thead>
<tr>
<th>Section</th>
<th>Area</th>
<th>( y )</th>
<th>( A_y )</th>
<th>( A_y^2 )</th>
<th>( I_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab 38x12x4</td>
<td>1,824</td>
<td>-2.00</td>
<td>-3,648</td>
<td>7,296</td>
<td>2,432</td>
</tr>
<tr>
<td>Beams 40x20</td>
<td>800</td>
<td>-14.00</td>
<td>-11,200</td>
<td>156,800</td>
<td>26,687</td>
</tr>
<tr>
<td>2,624</td>
<td>-14,848</td>
<td>164,096</td>
<td>29,099</td>
<td></td>
<td></td>
</tr>
<tr>
<td>29,099</td>
<td>193,195</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ Y_t = \frac{\Sigma A_y}{\Sigma A} = \frac{-14,848}{2,524} = -5.66 \text{ in.} \]

\[ I = (\Sigma A_y^2 + \Sigma I_o) - A_y^2 = 193,195 - (2,624 \times 5.66^2) \]

\[ I = 193,195 - 84,061 = 109,134 \text{ in.}^4 \]

\[ S_t = \frac{I}{Y_t} = \frac{109,134}{5.66} = 19,282 \text{ in.}^3 \]

\[ S_b = \frac{I}{Y_b} = \frac{109,134}{18.34} = 5,951 \text{ in.}^3 \]

**D. Prestressing Steel Requirements**

1. **Number of tendons required for minimum average prestress:**
   a. Stress in tendons immediately after anchoring:
   \[ f_p = 0.7 f_{pu} = 0.7 \times 270 = 189 \text{ ksi} \]

   b. Stress in tendons after losses (low-relaxation strand) assuming a lump sum value of 15 ksi for prestress losses. Actual losses should be calculated in accordance with section 6.6.:
   \[ f_b = 189 - 15 = 174 \text{ ksi} \]

   \[ N_{t(long)} = \frac{50 A_{long}}{f_{pA_{ps}}} = \frac{50}{1,000} = 50 \]

   Where \( A_{ps} = \) cross-sectional area of one tendon, in.\(^2\)

   \[ N_{t(long)} = \frac{(50 \text{ psi}) (2.624 \text{ in.}^2)}{1,000} = 4.9 \]

   \[ N_{t(long)} = \frac{(174 \text{ ksi}) (0.153)}{\text{in.}^2} = 5.1 \]

   \[ 2. \text{ Number of tendons required to overcome slab--subgrade friction (on polyethylene sheeting):} \]

   Weight of Beams and Slab = 62,223 + 76,000
   \[ = 138,223 \text{ lb.} \]

   \[ N_t = \frac{\mu W_{slab}}{2,000 f_{pA_{ps}}} \]
Where \( A_{ps} \) = cross-sectional area of one tendon, in.²

\[
N_i = \frac{0.75 \times 138,223}{2,000 \times 174 \times 0.153} = 1.95
\]

3. Total number of tendons to provide 50 psi minimum:

- **Long Direction**
  - \( N_T = 4.9 + 1.95 = 6.85 \) USE 9*
  - USE 9*

- **Short Direction**
  - \( N_T = 5.1 + 1.95 = 7.05 \) USE 9*

* Number of tendons has been increased so as to limit spacing to a maximum of 5'-0".

4. Design Prestress Force:

Force per tendon = \( f_p \times A_{ps} = 174(0.153) = 26.6 \) kips

\[
P_T = N_T(f_p \times A_{ps}) - \mu \left( \frac{W_{slab}}{2,000} \right)
\]

- **Long Direction**: \( P_T = 9(26.6) - \frac{0.75 \times 138,223}{2,000} = 187.57 \) kips

- **Short Direction**: \( P_T = 187.57 \) kips

Summary:

<table>
<thead>
<tr>
<th></th>
<th>Long Direction</th>
<th>Short Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross Sectional Area ( A ), (in.²)</td>
<td>2,624</td>
<td>2,720</td>
</tr>
<tr>
<td>Centroid of Strands, (in. from top)</td>
<td>-2.00</td>
<td>-2.00</td>
</tr>
<tr>
<td>Top Depth to Section Centroid ( y_1 ), (in.)</td>
<td>-5.66</td>
<td>-5.53</td>
</tr>
<tr>
<td>Prestress eccentricity, ( e ), (in.)</td>
<td>3.66</td>
<td>3.53</td>
</tr>
</tbody>
</table>

Allowable concrete tensile stress

\( f_c = 6\sqrt{3,000} = 328 \) psi = 0.328 ksi

Allowable concrete compressive stress

\( f_c = 0.45(3,000) = 1,350 \) psi = 1.35 ksi

E. Design Moments

1. **Long Direction**
   a. Moment for "no-swell" condition:

   \[
   M_{nsl} = \frac{h^{1.35}S^{0.36}}{60L^{0.12}p^{0.3}}
   \]

   \[
   M_{nsh} = \frac{24^{1.35}12.67^{0.36}}{80 \times 40^{0.12}940^{0.1}}
   \]

   \( M_{nsl} = 0.746 \) ft., kips/ft.

   b. Differential Deflection for "no-swell" condition:

   \[
   \Delta_{nsl} = \frac{L^{1.28}S^{0.82}}{133h^{0.28}p^{0.62}}
   \]

   \[
   \Delta_{nsh} = \frac{40^{1.28}12.67^{0.82}}{133 \times 24^{0.28}840^{0.52}} = 0.041
   \]

   \[
   M_{cal} = \left( \frac{A}{2} \right) \frac{M_{nsl}}{M_{nsh}}
   \]

   \[
   M_{cal} = \left( \frac{0.75}{0.041} \right) 0.746 = 3.19 \text{ ft. kips/ft.}
   \]

2. **Short Direction**
   a. Design Moment:

   \[
   M_{cal} = \left( \frac{970 - h}{880} \right)^{0.5} M_{cal}
   \]

   \[
   M_{cal} = \left( \frac{970 - 24}{880} \right) 3.19
   \]

   \( = 3.42 \) ft. kips/ft.

F. Compare Actual and Allowable Service Load Stresses

1. **Long Direction**
   a. Compression in top fiber (tension negative, compression positive):

   \[
   f = \frac{P_T}{A} + \frac{M_{cal}}{S_t} + \frac{P_e}{S_t}
   \]

   \[
   f = \frac{187.57 + 3.19 \times 38 \times 12 + 187.57 \times 3.66}{2,524} + \frac{19,282}{5,951} + \frac{19,282}{5,951}
   \]

   \( f = +0.183 \) ksi

   0.183 ksi < 1.125 ksi OK

   b. Tension in bottom fiber:

   \[
   f = \frac{P_T}{A} - \frac{M_{cal}}{S_b} - \frac{P_e}{S_b}
   \]

   \[
   f = \frac{187.57 - 3.19 \times 38 \times 12 - 187.57 \times 3.66}{2,524} - \frac{5,951}{5,951} - \frac{5,951}{5,951}
   \]

   \( f = -0.288 \) ksi < -0.328 ksi OK

2. **Short Direction**
   a. Compression in top fiber:

   \[
   f = \frac{P_T}{A} + \frac{M_{cal}}{S_t} + \frac{P_e}{S_t}
   \]

   \[
   f = \frac{187.57 + 3.42 \times 40 \times 12 + 187.57 \times 3.53}{2,720} + \frac{19,887}{19,887} + \frac{19,887}{19,887}
   \]

   \( f = +0.184 \) ksi

   0.184 ksi < 1.125 ksi OK
b. Tension in bottom fiber:
\[
f = \frac{P_z - M_S}{A_S} - \frac{P_x}{S_p} = \frac{187.57 - 3.42 \times 40 \times 12}{5,984} - \frac{187.57 \times 3.53}{5,984}
\]
\[f = -0.316 \text{ ksi} < -0.338 \text{ ksi} \quad \text{OK}
\]
Service Load bending stresses are OK.

G. Deflection Calculations (C_A = 1,920 from Table 6.2)

1. Long Direction
   a. Relative Stiffness Length:
   \[
   \Delta_{ns_L} = \frac{L^{1.28}S_{0.8}}{133h^{0.28}P^{0.62}} = 0.041
   \]
   \[
   \Delta_{ns_L} = \frac{12^{0.28}1.28^{0.8}}{133 \times 24^{0.28} \times 940^{0.62}} = 0.041
   \]

   \[
   \beta = \frac{1}{124} \frac{E_{IL}}{E_S} = \frac{1}{124} \frac{15 \times 10^6}{103,134} = \frac{15 \times 10^6}{1,000} \frac{0.75}{0.041}
   \]

   \[\beta = 4.56 \text{ ft.}
   \]
   \[L = 6\beta = 27.4 \text{ ft.} < 40 \text{ ft.}
   \]
Use 27.4 ft. as basis for deflection calculations.

b. Allowable Differential Deflection:
   \[
   \Delta_{allow} = \frac{12(L \text{ or } 6\beta)}{C_A} = \frac{27.4 \times 12}{1,920} = 0.17 \text{ in.}
   \]

   \[
   \Delta_{cs} = \frac{\delta}{E_S} \left[ 1.78 - 0.103h - 165x10^{-3}P + 3.95 \times 10^{-7}P^2 \right]
   \]
   \[\Delta_{cs} = 0.75e_n \left[ 1.78 - 0.103(24) - 165x10^{-3}(840) + 3.95 \times 10^{-7}(840)^2 \right]
   \]
   \[\Delta_{cs} = 0.124 \text{ in.}
   \]
   \[0.124 \text{ in.} < 0.17 \text{ in.} \quad \text{OK}
   \]
Deflection is OK in the long direction.

H. Shear Calculations

1. Long Direction
   a. Expected Service Shear:
   \[
   V_{nsL} = \frac{b^{0.9}(PS)^{0.3}}{550L^{0.1}} = 0.355 \text{ kips/ft.}
   \]
   \[
   V_{nsL} = \frac{24^{0.9}(840 \times 12.67)^{0.3}}{550 \times 40^{0.1}} = 0.355 \text{ kips/ft.}
   \]
   \[
   V_{csL} = \left( \frac{\delta}{E_S} \right)^{0.3} V_{nsL} = \left( \frac{0.75}{0.041} \right)^{0.3} = 0.355
   \]
   \[
   V_{csL} = 0.349 \text{ kips/ft.}
   \]

   b. Permissible Shear Stress:
   \[
   V_c = 1.7 \sqrt{f_c} + 0.2 \frac{187.7}{2,624}
   \]
   \[
   V_c = 93 + 14 = 107 \text{ psi}
   \]
   \[
   V = \frac{V_{csL} W}{nbh} = \frac{0.849(38)(1,000)}{4(10)(24)} = 34 \text{ psi} < 107 \text{ psi} \quad \text{OK}
   \]
Shear stress is OK in the long direction.

2. Short Direction
   a. Expected Service Shear:
      \[ V_{csa} = \left[ \frac{116 - h}{94} \right] V_{csa} \]
      \[ = \left[ \frac{116 - 24}{94} \right] 0.849 \]
      \[ = 0.831 \]
   b. Permissible Shear Stress:
      \[ v = 1.7\sqrt{f_c} + 0.2f_p \]
      \[ = 1.7\sqrt{3,000} + 0.2 \frac{187.5}{2720} \]
      \[ = 93 + 13 = 106 \text{ psi} \]
   c. Design (Actual) Shear Stress:
      \[ v = \frac{V_{csa} W}{nbh} = \frac{0.831(40)(1000)}{4(10)(24)} \]
      \[ v = 34 \text{ psi} < 106 \text{ psi} \quad \text{OK} \]

Shear stress is OK in the short direction.

Shear is OK in both directions.

A.8.3 Design Summary

A. Long Direction:
   Use 24" deep beams, 10" wide, spaced either 13'-0" or 14'-0" on center, nine 1/2"-270 ksl low-relaxation tendons in the slab with centroids 2" below top of slab and at 4'-3" on center, beginning 2'-0" from each edge (total of 4 beams and 9 tendons.)

B. Short Direction:
   Use 24" deep beams, 10" wide, spaced at 12'-8" on center. Use nine 1/2"-270 ksl low-relaxation tendons in the slab with the centroids 2" below top of slab and at 4'-6" on center beginning 2'-0" from each edge (total of 4 beams and 9 tendons.)