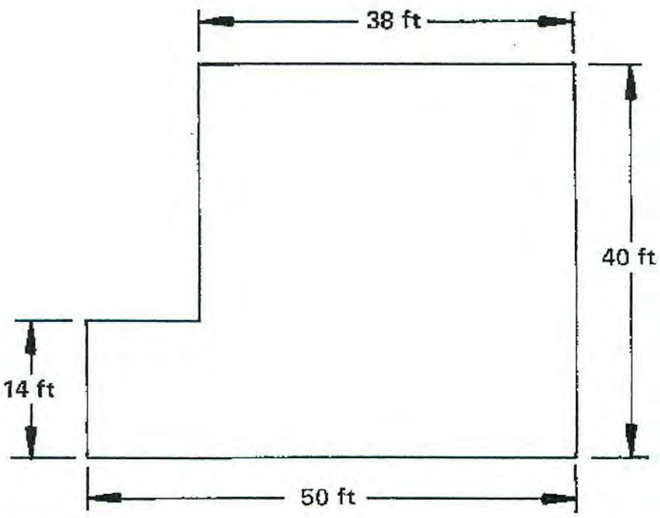
APPENDIX A.8
Design Example: Residential Slab on Compressible Soil

GIVEN: A single–story residence in Alexandria, Louisiana, with the dimensions as shown. Construction is wood frame with concrete masonry units for exterior walls and sheetrock interior, with foundations built on polyethylene sheeting.



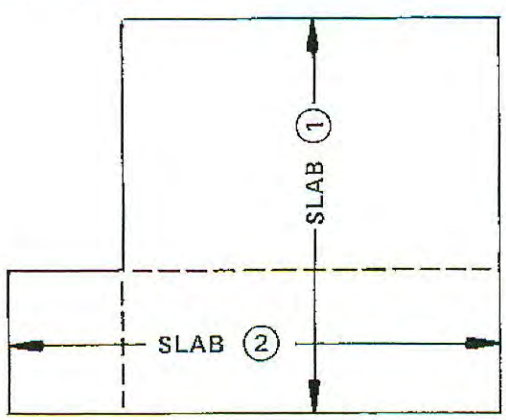
A.8.1. Design Data

A. Loading
1. No interior load–bearing partitions.
2. Perimeter loading **P** = 840 lb/ft.

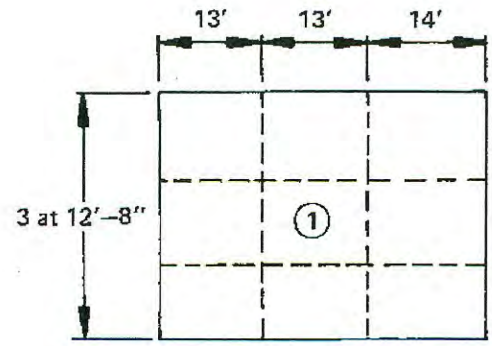
B. Materials
1. Concrete: f'_c = 3,000 psi
2. Prestressing steel – 1/2″ ϕ – 270 ksi 7–wire low-relaxation strand

C. Soils Investigation
1. Soil Type: CH
2. Expected soil settlement δ = 0.75 inch
3. Allowable soil bearing pressure q_{allow} = 1,500 psf

D. Separate floor plan into overlapping rectangles as shown below.



E. Design Slab ① only (for illustrative purposes). Assume spacing of stiffening beams as shown below.



A.8.2 Edge Lift Design (Predominant Distortion Mode)
A. Approximate Depth of Stiffening Beam

Note: Experience has shown that an initial assumption of e_m = 1 will yield a satisfactory trial section.

Assume:

e_m = 1.0 ft. (for initial estimate of beam depth only)
y_m = 0.75 inch (conservative since δ = 0.75 in.)
β = 4.0 ft.

1. Long Direction

C_Δ = 1,920 (Table 6.2)

L = 40 ft. > 6β = 6(4) = 24 ft. USE 24 ft.

$$\Delta_{allow} = \frac{12(L \text{ or } 6\beta)}{C_\Delta} = \frac{12 \times 24}{1{,}920} = 0.15 \text{ in.}$$

$$h = \left(\frac{(L)^{0.35} S^{0.88} e_m^{\ 0.74} y_m^{\ 0.76}}{15.9\Delta_{allow} P^{0.01}}\right)^{1.176}$$

$$h = \left(\frac{(40)^{0.35} 12.67^{0.88} 1.0^{0.74} 0.75^{0.76}}{15.9 \times 0.15 \times 840^{0.01}}\right)^{1.176} = 16.2 \text{ in.}$$

2. Short Direction

L = 38 ft. > 6β = 6(4) = 24 ft. USE 24 ft.

$$\Delta_{allow} = \frac{12 \times 24}{1{,}920} = 0.15 \text{ in.}$$

$$h = \left(\frac{(L)^{0.35} S^{0.88} e_m^{\ 0.74} y_m^{\ 0.76}}{15.9\Delta_{allow} P^{0.01}}\right)^{1.176}$$

$$h = \left(\frac{(38)^{0.35}13.33^{0.88}1.0^{0.74}0.75^{0.76}}{15.9 \times 0.15 \times 840^{0.01}}\right)^{1.176} = 16.8 \text{ in.}$$

Several iterations have been skipped for the sake of brevity.

Try h = 24 in.
b = 10 in.

B. Check Soil Bearing

1. Allowable Soil Pressure:

q_{allow} = 1,500 psf

2. Applied Loading:

Slab weight = 40x38x0.333x150	= 76,000 lb.
Added **DL** = 15x40x38	= 22,800 lb.
Beam weight = 20x10x298.67/144x150	= 62,223 lb.
Perimeter Load **P** = 156x840	= 131,040 lb.
Live Load = 40x38x40	= 60,800 lb.
Total	**352,863 lb.**

3. Beam Bearing Area = 298.67 ft x 0.833
= 248.8 sq. ft.

4. Soil Pressure = 352,863/248.8
= 1,418 psf < 1,500 psf OK

C. Section Properties

	Long Direction	Short Direction
Beam depth **h** (in.)	24	24
Beam Width (each) (in.)	10	10
Number of beams	4	4
Total Beam Width (in.)	40	40
Slab thickness (in.)4	4	

Long Direction

Section	*Area*	*y*	*Ay*	*Ay²*	*I₀*
Slab 38x12x4	1,824	−2.00	−3,648	7,296	2,432
Beams 40x20	800	−14.00	−11,200	156,800	26,667
	2,624		−14,848	164,096	29,099
				29,099	
				193,195	

$$y_t = \frac{\sum Ay}{\sum A} = \frac{-14,848}{2,624} = -5.66 \text{ in.}$$

$$I = (\sum Ay^2 + \sum I_o) - Ay_t^2 = 193,195 - (2,624 \times 5.66^2)$$

$$I = 193,195 - 84,061 = 109,134 \text{ in.}^4$$

$$S_t = \frac{I}{y_t} = \frac{109,134}{5.66} = 19,282 \text{ in.}^3$$

$$S_b = \frac{I}{y_b} = \frac{109,134}{18.34} = 5,951 \text{ in.}^3$$

Short Direction

Section	*Area*	*y*	*Ay*	*Ay²*	*I₀*
Slab 40x12x4	1,920	−2.00	−3,840	7,680	2,560
Beams 40x20	800	−14.00	−11,200	156,800	26,667
	2,720		−15,040	164,480	29,227
				29,227	
				193,707	

$$y_t = \frac{\sum Ay}{\sum A} = \frac{-15,040}{2,720} = -5.53 \text{ in.}$$

$$I = (\sum Ay^2 + \sum I_o) - Ay_t^2 = 193,707 - (2,720 \times 5.53^2)$$

$$I = 193,707 - 83,180 = 110,527 \text{ in.}^4$$

$$S_t = \frac{I}{y_t} = \frac{110,527}{5.53} = 19,987 \text{ in.}^3$$

$$S_b = \frac{I}{y_b} = \frac{110,527}{18.47} = 5,984 \text{ in.}^3$$

D. Prestressing Steel Requirements

1. Number of tendons required for minimum average prestress:

a. Stress in tendons immediately after anchoring:

$$f_{pi} = 0.7f_{pu} = 0.7(270) = 189 \text{ ksi}$$

b. Stress in tendons after losses (low–relaxation strand) assuming a lump sum value of 15 ksi for prestress losses. Actual losses should be calculated in accordance with section 6.6.:

$$f_e = 189 - 15 = 174 \text{ ksi}$$

$$N_{t(long)} = \frac{50A_{long} / 1,000}{f_e A_{ps}}$$

Where A_{ps} = cross-sectional area of one tendon, in.^{2}

$$N_{t(long)} = \frac{(50 \text{ psi})(2,624 \text{ in.}^2) / 1,000}{(174 \text{ ksi})\left(0.153 \frac{\text{in.}^2}{\text{strand}}\right)} = 4.9$$

$$N_{t(short)} = \frac{(50 \text{ psi})(2,720 \text{ in.}^2) / 1,000}{(174 \text{ ksi})\left(0.153 \frac{\text{in.}^2}{\text{strand}}\right)} = 5.1$$

2. Number of tendons required to overcome slab–subgrade friction (on polyethylene sheeting):

Weight of Beams and Slab = 62,223 + 76,000
= 138,223 lb.

$$N_t = \frac{\mu W_{slab}}{2,000 f_e A_{ps}}$$

Where A_{ps} = cross-sectional area of one tendon, in.^{2}

$$N_t = \frac{0.75 \times 138{,}223}{2{,}000 \times 174 \times 0.153} = 1.95$$

3. Total number of tendons to provide 50 psi minimum:

   Long $\mathbf{N_T} = 4.9 + 1.95 = 6.85$ USE 9*
   Short $\mathbf{N_T} = 5.1 + 1.95 = 7.05$ USE 9*

   * Number of tendons has been increased so as to limit spacing to a maximum of 5′–0″.

4. Design Prestress Force:

   Force per tendon = $\mathbf{f_e} \times \mathbf{A_{ps}} = 174(0.153) = 26.6$ kips

$$\mathbf{P_r} = N_T(f_e \times A_{ps}) - \mu\left(\frac{W_{slab}}{2{,}000}\right)$$

Long: $\mathbf{P_r} = 9(26.6) - \frac{0.75\ (138{,}223)}{2{,}000} = 187.57$ kips

Short: $\mathbf{P_r} = 187.57$ kips

Summary:

	Long Direction	*Short Direction*
Cross Sectional Area **A**, (in.^{2})	2,624	2,720
Centroid of Strands, (in. from top)	–2.00	–2.00
Top Depth to Section Centroid $\mathbf{y_t}$, (in.)	–5.66	–5.53
Prestress eccentricity, **e** (in.)	3.66	3.53
Allowable concrete tensile stress $f_t = 6\sqrt{3{,}000} = 328$ psi $= 0.328$ ksi	0.328	0.328
Allowable concrete compressive stress $f_c = 0.45(3{,}000) = 1{,}350$ psi $= 1.35$ ksi	1.35	1.35

E. Design Moments

1. Long Direction

   a. Moment for "no–swell" condition:

$$M_{ns_L} = \frac{h^{1.35}S^{0.36}}{80L^{0.12}P^{0.1}}$$

$$M_{ns_L} = \frac{24^{1.35}12.67^{0.36}}{80 \times 40^{0.12}840^{0.1}}$$

$$M_{ns_L} = 0.746 \text{ ft. kips / ft.}$$

   b. Differential Deflection for "no–swell" condition:

$$\Delta_{ns_L} = \frac{L^{1.28}S^{0.80}}{133h^{0.28}P^{0.62}}$$

$$\Delta_{ns_L} = \frac{40^{1.28}12.67^{0.80}}{133 \times 24^{0.28}840^{0.62}} = 0.041$$

   c. Design Moment:

$$M_{cs_L} = \left(\frac{\delta}{\Delta_{ns_L}}\right)^{0.5} M_{ns_L}$$

$$M_{cs_L} = \left(\frac{0.75}{0.041}\right)^{0.5} 0.746 = 3.19 \text{ ft. kips / ft.}$$

2. Short Direction

   a. Design Moment:

$$M_{cs_S} = \left(\frac{970 - h}{880}\right)^{0.5} M_{cs_L}$$

$$M_{cs_S} = \left(\frac{970 - 24}{880}\right)3.19$$

$$= 3.42 \text{ ft. kips / ft.}$$

F. Compare Actual and Allowable Service Load Stresses

1. Long Direction

   a. Compression in top fiber (tension negative, compression positive):

$$f = \frac{P_r}{A} + \frac{M_L}{S_t} + \frac{P_r e}{S_t}$$

$$f = \frac{187.57}{2{,}624} + \frac{3.19 \times 38 \times 12}{19{,}282} + \frac{187.57 \times 3.66}{19{,}282}$$

$$f = +0.183 \text{ ksi}$$

   0.183 ksi < 1.125 ksi OK

   b. Tension in bottom fiber:

$$f = \frac{P_r}{A} - \frac{M_L}{S_b} - \frac{P_r e}{S_b}$$

$$f = \frac{187.57}{2{,}624} - \frac{3.19 \times 38 \times 12}{5{,}951} - \frac{187.57 \times 3.66}{5{,}951}$$

$$f = -0.288 \text{ ksi} < -0.328 \text{ ksi} \quad \text{OK}$$

2. Short Direction

   a. Compression in top fiber:

$$f = \frac{P_r}{A} + \frac{M_S}{S_t} + \frac{P_r e}{S_t}$$

$$f = \frac{187.57}{2{,}720} + \frac{3.42 \times 40 \times 12}{19{,}987} + \frac{187.57 \times 3.53}{19{,}987}$$

$$f = +0.184 \text{ ksi}$$

   0.184 ksi < 1.125 ksi OK

b. Tension in bottom fiber:

$$f = \frac{P_r}{A} - \frac{M_S}{S_b} - \frac{P_r e}{S_b}$$

$$f = \frac{187.57}{2{,}720} - \frac{3.42 \times 40 \times 12}{5{,}984} - \frac{187.57 \times 3.53}{5{,}984}$$

$$f = -0.316 \text{ ksi} < -0.328 \text{ ksi} \qquad \text{OK}$$

Service Load bending stresses are OK.

G. Deflection Calculations (C_Δ = 1,920 from Table 6.2)

1. Long Direction

a. Relative Stiffness Length:

$$\Delta_{ns_L} = \frac{L^{1.28}S^{0.80}}{133h^{0.28}P^{0.62}}$$

$$\Delta_{ns_L} = \frac{40^{1.28}12.67^{0.80}}{133 \times 24^{0.28}840^{0.62}} = 0.041$$

$$\beta = \frac{1}{12}\sqrt[4]{\frac{E_cI}{E_s\left(\frac{\delta}{\Delta_{ns}}\right)}} = \frac{1}{12}\sqrt[4]{\frac{(1.5\text{x}10^6)(109{,}134)}{1{,}000\left(\frac{0.75}{0.041}\right)}}$$

$$\beta = 4.56 \text{ ft.}$$

$$L = 6\beta = 27.4 \text{ ft.} < 40 \text{ ft.}$$

Use 27.4 ft. as basis for deflection calculations.

b. Allowable Differential Deflection:

$$\Delta_{allow} = \frac{12(L \text{ or } 6\beta)}{C_\Delta} = \frac{27.4 \times 12}{1{,}920} = 0.17 \text{ in.}$$

c. Expected Differential Deflection:

$$\Delta_{cs} = \delta e_n^{[1.78 - 0.103h - 1.65\text{x}10^{-3}P + 3.95 \times 10^{-7}P^2]}$$

$$\Delta_{cs} = 0.75e_n^{[1.78 - 0.103(24) - 1.65\text{x}10^{-3}(840) + 3.95 \times 10^{-7}(840)^2]}$$

$$\Delta_{cs} = 0.124 \text{ in.}$$

0.124 in. < 0.17 in. OK

Deflection is OK in the long direction.

1. Short Direction

a. Relative Stiffness Length:

$$\Delta_{ns_S} = \frac{L^{1.28}S^{0.8}}{133h^{0.28}P^{0.62}}$$

$$\Delta_{ns_S} = \frac{38^{1.28}13.33^{0.8}}{133 \times 24^{0.28} \times 840^{0.62}} = 0.04$$

$$\beta = \frac{1}{12}\sqrt[4]{\frac{E_cI}{E_s\left(\frac{\delta}{\Delta_{ns}}\right)}} = \frac{1}{12}\sqrt[4]{\frac{(1.5\text{x}10^6)(110{,}527)}{1{,}000\left(\frac{0.75}{0.040}\right)}}$$

$$\beta = 4.54 \text{ ft.}$$

$$L = 6\beta = 27.2 \text{ ft.} < 40 \text{ ft.}$$

Use 27.2 ft. as basis for deflection calculations.

b. Allowable Differential Deflection:

$$\Delta_{allow} = \frac{12(L \text{ or } 6\beta)}{C_\Delta} = \frac{27.2 \times 12}{1{,}920} = 0.17 \text{ in.}$$

c. Expected Differential Deflection:

$$\Delta_{cs} = \delta e_n^{[1.78 - 0.103h - 1.65\text{x}10^{-3}P + 3.95 \times 10^{-7}P^2]}$$

$$\Delta_{cs} = 0.75e_n^{[1.78 - 0.103(24) - 1.65\text{x}10^{-3}(840) + 3.95 \times 10^{-7}(840)^2]}$$

$$\Delta_{cs} = 0.124 \text{ in.}$$

0.124 in. < 0.17 in. OK

Deflection is OK in the short direction.

H. Shear Calculations

1. Long Direction

a. Expected Service Shear:

$$V_{ns_L} = \frac{h^{0.9}(PS)^{0.3}}{550L^{0.1}}$$

$$V_{ns_L} = \frac{24^{0.9}(840 \times 12.67)^{0.3}}{550 \times 40^{0.1}} = 0.355 \text{ kips / ft.}$$

$$V_{cs_L} = \left(\frac{\delta}{\Delta_{ns}}\right)^{0.3} V_{ns_L} = \left(\frac{0.75}{0.041}\right)^{0.3} 0.355$$

$$V_{cs_L} = 0.849 \text{ kips / ft.}$$

b. Permissible Shear Stress:

$$v_c = 1.7\sqrt{f'_c} + 0.2f_p$$

$$v_c = 1.7\sqrt{3{,}000} + 0.2\frac{187.7}{2{,}624}$$

$$v_c = 93 + 14 = 107 \text{ psi}$$

c. Design (Actual) Shear Stress:

$$v = \frac{V_{cs_L}W}{nbh} = \frac{0.849(38)(1{,}000)}{4(10)(24)}$$

$$v = 34 \text{ psi} < 107 \text{ psi} \qquad \text{OK}$$

Shear stress is OK in the long direction.

2. Short Direction

a. Expected Service Shear:

$$V_{cs_S} = \left[\frac{116 - h}{94}\right] V_{cs_L}$$

$$= \left[\frac{116 - 24}{94}\right] 0.849$$

$$= 0.831$$

b. Permissible Shear Stress:

$$v_c = 1.7\sqrt{f'_c} + 0.2f_p$$

$$= 1.7\sqrt{3{,}000} + 0.2\frac{187.5}{2{,}720}$$

$$= 93 + 13 = 106 \text{ psi}$$

c. Design (Actual) Shear Stress:

$$v = \frac{V_{cs_S} W}{nbh} = \frac{0.831(40)(1{,}000)}{4(10)(24)}$$

$v = 34$ psi $<$ 106 psi OK

Shear Stress is OK in the short direction.

Shear is OK in both directions.

A.8.3 Design Summary

A. Long Direction:

Use 24″ deep beams, 10″ wide, spaced either 13′–0″ or 14′–0″ on center, nine 1/2″–270 ksi low-relaxation tendons in the slab with centroids 2″ below top of slab and at 4′–3″ on center, beginning 2′–0″ from each edge (total of 4 beams and 9 tendons.)

B. Short Direction:

Use 24″ deep beams, 10″ wide, spaced at 12′–8″ on center. Use nine 1/2″–270 ksi low-relaxation tendons in the slab with the centroids 2″ below top of slab and at 4′–6″ on center beginning 2′–0″ from each edge (total of 4 beams and 9 tendons.)