Closure to Discussion by Kenneth B. Bondy concering "*Monitoring Secondary Moments of Continuous Unbonded Post-Tensioned Concrete Beams*," Kyungmin Kim and Thomas H.-K. Kang, *PTI Journal*, December 2018, pp. 5-16

BY KYUNGMIN KIM AND THOMAS H.-K. KANG
WITH CONTRIBUTION FROM HYEONGYEOP SHIN AND BYEONGUK AHN, SEOUL NATIONAL UNIVERSITY FOR INDEPENDENT DETAILED ANALYTICAL CALCULATIONS

APPENDIX—SECONDARY MOMENT CALCULATION USING INDETERMINATE FRAME ANALYSIS AND LOAD-BALANCING METHOD

In post-tensioned members with unbonded parabolic tendons, the balanced loads from PT tendons to concrete were calculated as shown in Fig. A1.

Example:

Numerical calculations for Specimen 4L. (Only SI units are provided for clarity.)

Specimen 4L consists of four different parabolas (refer to Fig. A2); thus, four different balanced uniform loads (w_1, w_2, w_3, and w_4) were calculated in similar manner to that in Fig. A1.

Equivalent loads for each tendon profile were calculated as follows:

Actual tendon profile (4L specimen):

$$w_1 = \frac{8Pa_1}{(2l_1)^2} = \frac{8(100)(5.19)}{(2\times151.67)^2} = 45.123 \text{ kN/m} \quad \text{(downward)} \tag{A1-1}$$

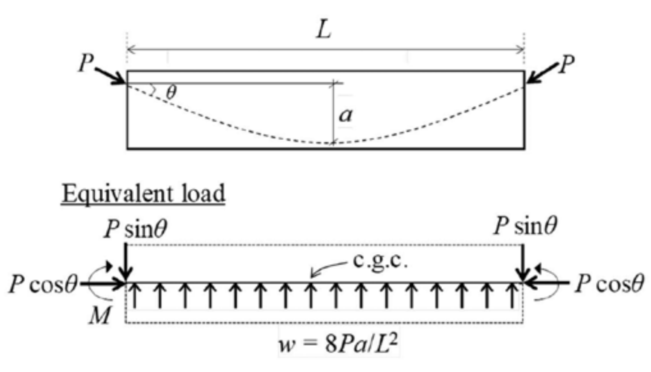


Fig. A1—Equivalent loads produced by parabolic post-tensioning tendon to concrete. (Note: P *is post-tensioning force;* L *is horizontal distance between anchorages;* a *is average vertical distance between anchorage level and lowest point of tendon profile; and* M *is equivalent end moment due to post-tensioning and eccentricity, applied to plain concrete.)*

$$w_2 = \frac{8Pa_2}{(2l_2)^2} = \frac{8(100)(82.81)}{(2\times2{,}419.13)^2} = 2.830 \text{ kN/m} \quad \text{(upward)} \tag{A1-2}$$

$$w_3 = \frac{8Pa_3}{(2l_3)^2} = \frac{8(100)(142.15)}{(2\times3{,}456.23)^2} = 2.380 \text{ kN/m} \quad \text{(upward)} \tag{A1-3}$$

$$w_4 = \frac{8Pa_4}{(2l_4)^2} = \frac{8(100)(33.85)}{(2\times822.97)^2} = 9.996 \text{ kN/m} \quad \text{(downward)} \tag{A1-4}$$

Idealized (ω-shaped) tendon profile:

$$w_1 = \frac{8Pa_1}{(2l_1)^2} = \frac{8(100)(88)}{(2\times2{,}570.80)^2} = 2.663 \text{ kN/m} \quad \text{(upward)} \tag{A2-1}$$

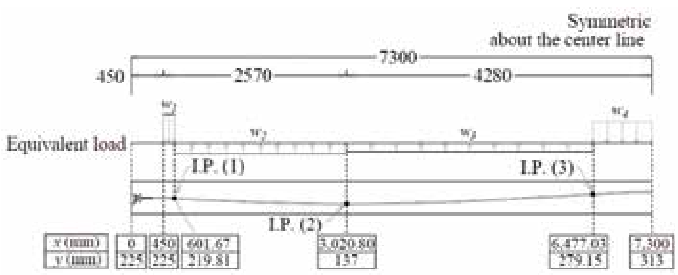
$$w_2 = \frac{8Pa_2}{(2l_2)^2} = \frac{8(100)(176)}{(2\times4{,}279.20)^2} = 1.922 \text{ kN/m} \quad \text{(upward)} \tag{A2-2}$$

$$F_1 = P\sin\theta_1 \simeq P\tan\theta_1 = w_1 l_1 = 6.846 \text{ kN} \quad \text{(downward)} \tag{A2-3}$$

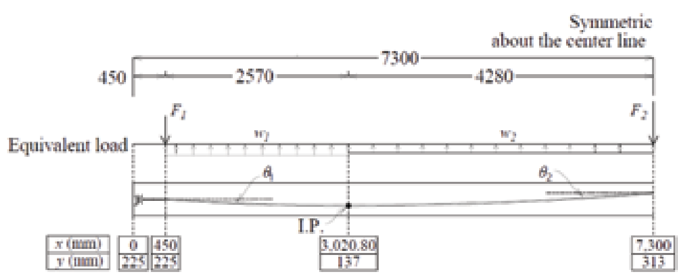
$$F_2 = 2P\sin\theta_2 \simeq 2P\tan\theta_2 = 2w_2 l_2 = 16.449 \text{ kN} \quad \text{(downward)} \tag{A2-4}$$

For elastic finite element analysis using ETABS, a fixed post-tensioning force of 100 kN was assumed for applied loads. Different equivalent loads per span were input as shown in Fig. A3 and A4 for given gross concrete properties.

Using linear analysis, secondary reactions were obtained as shown in Fig. A3 and A4, along with balanced moments (including both primary moment and secondary moment).



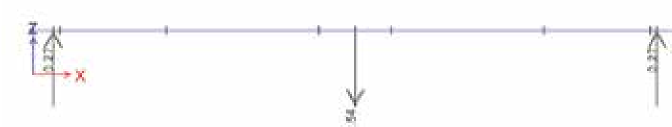
(a) Four different parabolas and equivalent loads for actual 4L specimen tendon profile



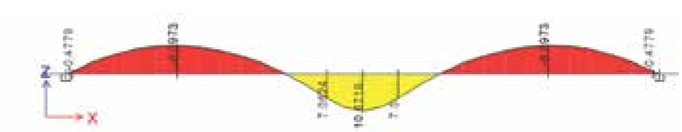
(b) Two different parabolas and equivalent loads for idealized (ω-shaped) tendon profile

Fig. A2—Tendon profile and equivalent uniformly distributed loads. (Note: Numbers in boxes give horizontal distance from left end and vertical height of tendon from beam bottom.)

(a) Equivalent loads (w_1, w_2, w_3 and w_4) to concrete



(b) Reaction at the supports



(c) Balanced moment (M_{bal})

Fig. A3—Equivalent loads applied to concrete and resulting balanced moment (actual tendon profile).

Given the M_{bal}, secondary moment can be calculated by using Eq. (A3) (indirect method).

$$M_{2\_calc} = M_{bal} - M_1 \quad \text{(A3)}$$

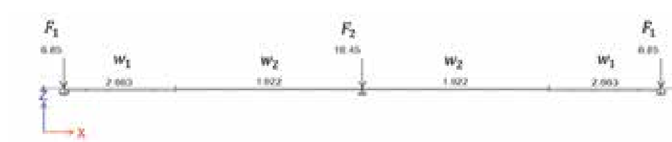
Where: M_{2_calc} is the calculated secondary moment in the beam, Mbal is the balanced moment (refer to Fig. 3A(c)), and M1 is the primary moment. Primary moment can be calculated using Eq. (A4)

$$M_1 = P_e \quad \text{(A4)}$$

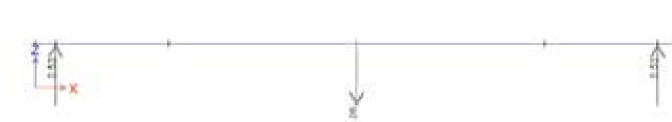
where P is the post-tensioning force at the section of interest (= 100 kN for this specimen); and e is the vertical distance between c.g.c. and c.g.s at the section of interest. (No friction is considered herein.)

Calculated secondary moments are shown in Table A1.

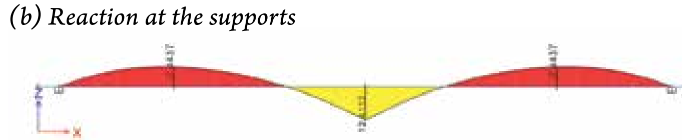
Note: The discrepancy between the secondary moment values based on the actual and idealized tendon profiles is approximately 93%, though the discrepancy between the peak balanced moment values at the interior support is approximately 16% (10.67 kN-m versus 12.41 kN-m; refer to Fig. A3 versus A4).



(a) Equivalent loads (w_1, w_2, F_1 and F_2) to concrete



(b) Reaction at the supports



(c) Balanced moment (M_{bal})

Fig. A4—Equivalent loads applied to concrete and resulting balanced moment (idealized tendon profile).

Table A1—Calculated secondary moments

Actual tendon profile (4L specimen)	Idealized (ω-shaped) tendon profile
0.0187P (kN-m)	0.0361P (kN-m)

P = post-tensioning force