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By

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POST-TENSIONING INSTITUTE ®

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ANALYSIS OF SHORTENING OF POST-TENSIONED SLABS

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Post-tensioned slab shortening is caused by shrinkage, elastic and creep deformations of concrete from prestress, and temperature drops. Shortening occurs during the construction and service life of the structure. Restraint forces reduce the pre-compression in slabs and may cause cracking in columns, walls, and slabs. Restraint forces are reduced by concrete relaxation. The prestressing force in slabs is responsible for a small portion of the restraint forces in the long term, because they are relaxed. The shrinkage is more important in term of displacements and forces. In open structures, restraint forces created by sudden temperature drops may be critical because they are not relaxed.

A simple analysis method that uses the equivalent moduli of elasticity from creep and shrinkage prediction models is presented. It reduces the time effects analysis into an instantaneous analysis using elastic structural analysis programs. Results highlight the need for mitigation measures. This analysis method is compared with the method given in the PCI Design Handbook *and the time-dependent analysis from a commercial program.*

KEYWORDS

concrete; creep; durability; post-tensioned slabs; prestress; restraint; shrinkage; slab shortening; temperature drops.

INTRODUCTION

ACI 318R-14[1] requires volume changes to be accounted for in the design but provides no means of analysis. In practice, it is taken into account with temporary and/or permanent separation details and rarely designed through analysis. According to ACI 362-97,[2] volume changes are the second cause, after corrosion, of structural problems in parking structures. It is also an important cause of damage in other structures. Shortening of post-tensioned (PT) slabs raised interest in papers published in *Concrete International*,[3,4] the PTI *JOURNAL*,[5] and *STRUCTURE* magazine.[6] Slab shortening was studied conceptually in depth by Aalami and Barth[7] and in the analysis method presented in the *PCI Design Handbook*[8] assessed by Klein and Lindenberg.[9] These references discuss some of the various slab shortening aspects such as tolerances, slab cracking, pour strips, restraint forces, moments in columns, and slabs-on-ground. An analysis method is provided to estimate the creep, shrinkage, and temperature effects on structures with post-tensioned slabs, incorporating the advances in creep and shrinkage prediction models and computer analysis capabilities. PCI reduction factors are also discussed. Results from the analysis of a structure with PT slabs using three different methods are compared.

RESEARCH SIGNIFICANCE

This paper provides updated approaches to the analysis of the effects of post-tensioned slab shortening. It summarizes, improves, and develops analysis methods from the simplest to the more complex for the evaluation of the reduction of forces and moments from slab shortening due to the concrete relaxation. A finding is that according to the Bažant-Baweja B3 (B3 model[11]), the maximum forces and moments occur in the early years of the structure, as observed in cracking development, while with other models forces keep increasing with time. Shrinkage and temperature shortening is the main cause of forces and moments compared with those from the elastic and creep shortening.

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MECHANISMS AND CONCEPTS

Design requirements

Volume changes are considered in the ACI 318-14[1] design requirement in Section 5.3.6. However, no load factor or combination are provided. It is mentioned that self-straining forces are not calculated in general, but rather treated with mitigation measures.

Basic assumptions

With the theory of aging linear viscoelasticity, the principle of superposition and the effective rheological homogeneity of the structure are consistently assumed for the analysis of the shortening of post-tensioned slabs in structures.

Slab shortening

There are four causes of slab shortening: elastic (EL) and creep (CR) from PT, and shrinkage (SH) and temperature (T). Figure 1 illustrates the shortening of an unrestrained PT slab, while Fig. 1.1 from ACI 209.1R-05[10] illustrates the elastic, creep, and shrinkage strains and their components.

Concrete creep and shrinkage depend on many factors, such as the concrete constituent materials and mixture proportions, load duration, age at loading, the shape and dimensions of the elements, and environmental conditions (curing, relative humidity, and temperature).

On the precision of the different prediction models ACI 209.2R-08[11] states that: "The variability of shrinkage and creep test measurements prevents models from closely matching experimental data. The within-batch coefficient of variation for laboratory-measured shrinkage on a single mixture of concrete was approximately 8% (Bažant et al. 1987). Hence, it would be unrealistic to expect results from prediction models to be within plus or minus 20% of the test data for shrinkage. Even larger differences occur for creep predictions." However, this precision may be enough to determine if slab shortening mitigation measures are needed. The precision can be greatly improved with short duration creep and shrinkage tests.[11]

Critical forces and moments from slab shortening may occur from a sudden temperature drop. A temperature increase provides relief to the structure shortening effects. Daily, weekly, and seasonal temperature drops must be considered in analysis.

Figure 2 shows the contributions to slab shortening in two structures with the same geometry as presented in the section "Prediction comparisons." They differ in the effective prestress of 1.034 MPa (150 psi) and 1.724 MPa (250 psi) in ambient relative humidity of 50% and 70%, and in the design temperature drop of –10 Δ°C (–18 Δ°F) and –40 Δ°C (–72 Δ°F), respectively. Shrinkage and temperature changes are the main causes of localized cracking in post-tensioned slabs and supporting members.

Relaxation of concrete

While creep is the increase in strain under a sustained stress, relaxation is the decrease in stress under a constant imposed strain. Creep and relaxation are two aspects of the same viscoelastic behavior of concrete. Time-induced

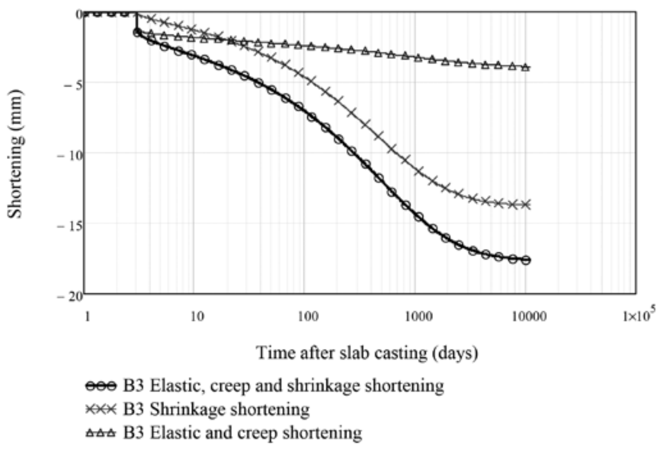

Fig. 1—Example of creep and shrinkage shortening of unrestrained slab using B3 model[10] up to 30 years. (Note: 1 mm = 0.039 in.)

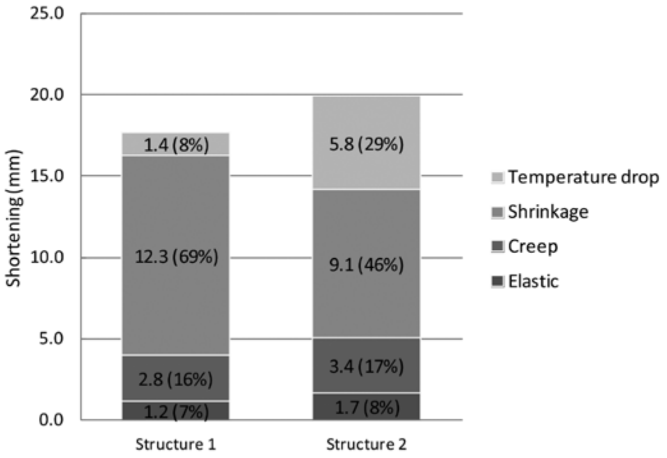

Fig. 2—Examples of contribution to slab shortening at 30 years from elastic, creep, and shrinkage strains and temperature drop. (Note: 1 mm = 0.039 in.)

restraint forces are relaxed in structural concrete elements and must be accounted for when analyzing the long-term behavior of a concrete structure.

In unrestrained slabs, there is no stress relaxation of stress from imposed forces such as prestress but creep deformation. In slabs continuous with supports, there is some concrete relaxation of stresses in PT slabs from the reduction of slab shortening due to columns and walls restraints; however, this is a secondary effect.

Time relaxation curves are usually presented for a given instantaneous imposed strain at time t_0. Figure 3 shows the relaxation predictions at different ages from the B3 model[11] and the CEB M90[11] model for $f_c' = 34.5$ MPa (5000 psi) concrete. This figure shows both the quantitative and behavioral differences between the models, especially when strains are imposed on old concrete. There is a difference at time of loading, but most importantly, the B3 model converges to zero with time while the CEBM90 is asymptotic.

Because restraint forces are produced and relaxed with time, the incremental loading age is the variable used for the evaluation of these forces at a given time from shrinkage and slow drops in temperature, as shown in Fig. 4. This aspect of the relaxation function requires an

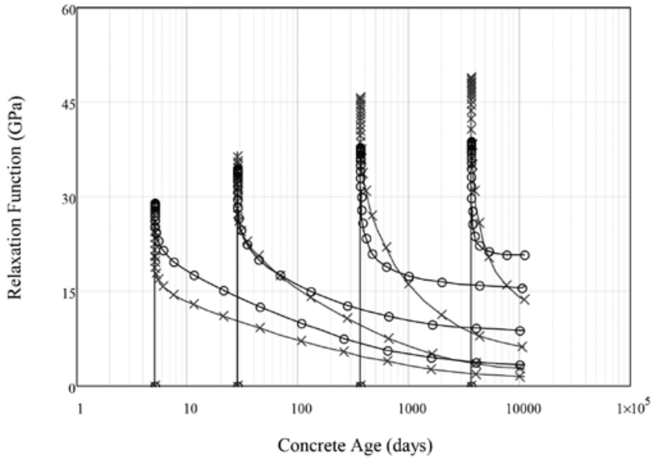

Fig. 3—Relaxation functions up to 30 years for imposed deformation at 5 days, 1 month, 1 year, and 10 years from B3 (× symbol) and CEB M90 (○ symbol) models. (Note: 1 GPa = 145 ksi.)

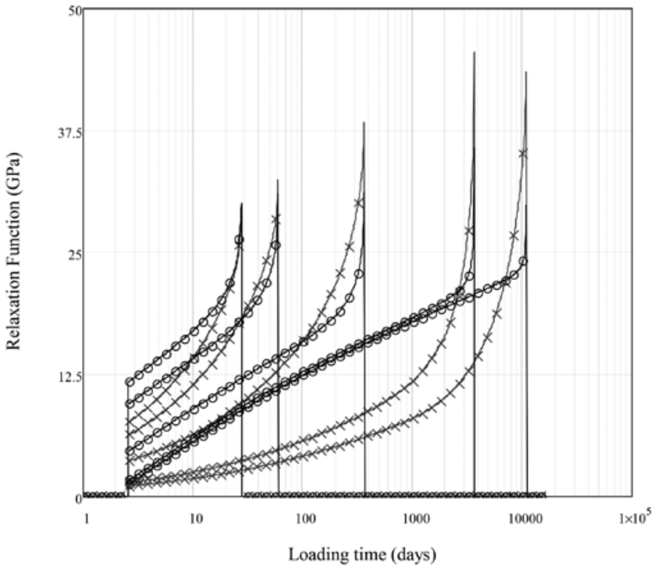

Fig. 4—Relaxation functions at 1 month, 2 months, 1 year, 10 years, and 30 years from B3 (× symbol) and CEB M90 (○ symbol) models. (Note: 1 GPa = 145 ksi.)

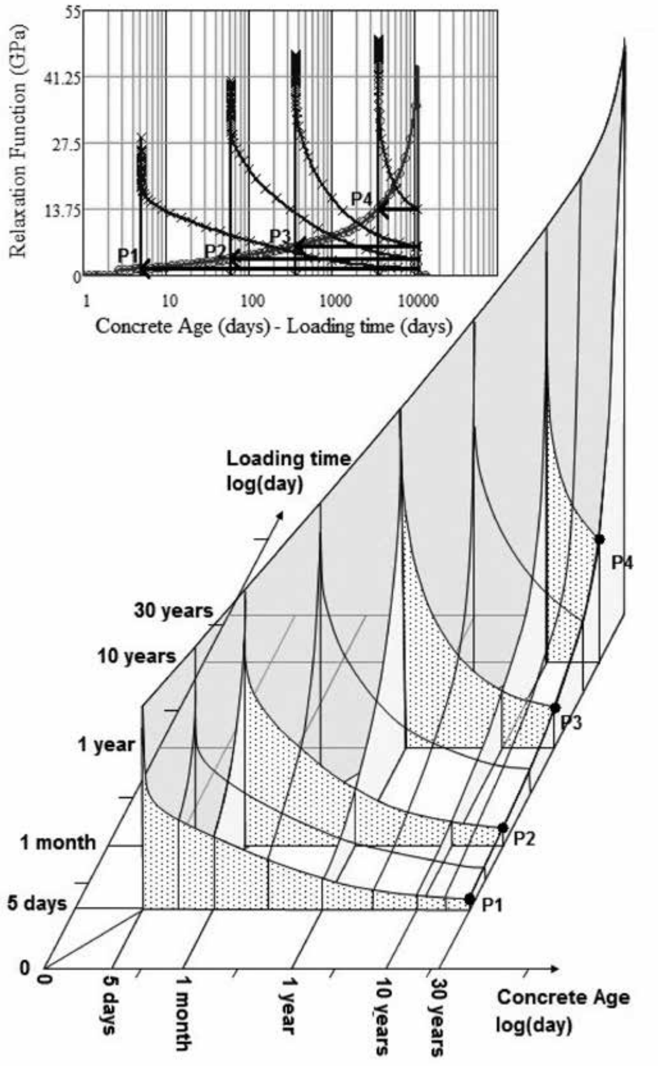

Fig. 5—3-D view of B3 relaxation function in log time scale and illustration of how it is built. (Note: 1 GPa = 145 ksi.)

elaborate computation compared to simpler creep strain calculations. The relationship between Figures 3 and 4 is explained in both the two-dimensional (2-D) and the three-dimensional (3-D) relaxation function, $R(t,t_0)$ in Fig. 5. Points P1, P2, P3, and P4 are the relaxation of a 30-year-old concrete to imposed displacements when it was 5 days, 1 month, 1 year, and 10 years old, respectively.

Restraint to shortening

Unrestrained slab shortening does not create forces. In most cases, foundations are the main source of restraint in structures, but they are not the only ones. The types of foundations and column and wall connections to foundations have a huge impact on the effects of the slab shortening. For instance, slabs-on-ground and foundation beams are the most critical restraints because they increase the rotational and displacement stiffness of foundations.

Depending on the soil, type of foundation (footings, caissons), beams in foundations, and slabs-on-ground, the column and wall connection can behave from fixed in displacement and rotation to lesser degrees of fixity. For instance, foundations on rock connected with beams are very close to being fixed in displacement and rotation.

The *PCI Design Handbook*[8] provides a method to compute the rotational stiffness from the allowable soil bearing stress value, but it is limited to isolated footings and displacements are not considered. Reduction in the foundation rotational stiffness may reduce restraint forces significantly in columns and in walls parallel to the shortening direction. Walls parallel to the shortening direction can be considered fixed unless the foundation is very flexible. For 3-D analyses, foundation rotational stiffness in each principal direction must be input.

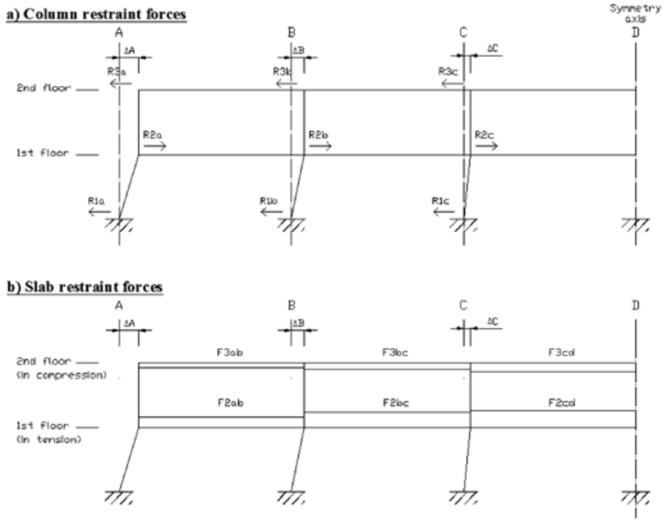

Fig. 6—Restraint forces in columns and slabs.

Slabs located below and above a transfer plate or transfer beams can be significantly restrained. An existing structure can provide considerable restraint to an addition built on top of it.[7] When designing additional floors on existing structures, this type of restraint must be accounted for in the design of the addition to the structure.

Damage induced by slab shortening

Partition walls, façades, and installations—Slab shortening can cause damage to nonstructural elements affected by displacements such as façades, partition walls, and installations.

Cracking in columns and walls from induced moments and shear forces—Slab shortening pulls columns and walls toward the structure stiffness center, creating shear forces and moments. Shear forces and moments on columns and walls from slab shortening are proportional to their stiffness and distance to the floor stiffness center, as shown in Fig. 6. Two opposite mechanisms occur simultaneously in the structure: slab shortening generates forces and moments over time that are partially relaxed by concrete. Restraint forces' evolution depends of the time rates of the creep and of shrinkage prediction models.

Slab shortening effects are especially important in exterior and corner columns because they experience the maximum shortening. Columns and walls may crack in flexure, in shear, or both, and may become heavily damaged.

Cracking and losses of pre-compression in prestressed slabs—Schematically, the frames in Fig. 6 show the restraint forces in columns and the restraint forces in the slabs, as in Reference 8. Equations (1) and (2) illustrate how the PCI Design Aids propose to compute the build-up of restraint forces in the slab toward its center. Forces in other spans and levels are computed similarly.

$$F_{2.CD} = \frac{k_{F2}(\Delta_A + \Delta_B + \Delta_C)\cdot E_C \cdot I_{col}}{h_s^3} \qquad (1)$$

$$\underset{\text{(time)}}{\textbf{Prestress}} + \underset{\text{(time, floor level, span)}}{\textbf{Stress changes from restraint forces}} = \underset{\text{(time, floor level, span)}}{\textbf{Resultant stresses}} \qquad (2)$$

Tension forces from restraints on the first-floor slab can drastically decrease the slab prestress and result in net tension in spans close to the stiffness center. Stiff walls on the slab perimeter or ramps can create significant tension forces over several floors. However, the loss of axial precompression in concrete does not decrease the balancing vertical force from the tendons or the flexural capacity of the slab.[7] Although cracks may not reduce the flexural capacity of the slabs, those near the columns and walls may reduce their shear capacity. Cracks may reduce the durability of the structure because they undermine the reinforcement's corrosion protection. This is critical in structures exposed to aggressive environments or deicing salts, such as parking structures or marine structures.

AVAILABLE ANALYSIS METHODS

There are several analysis methods of the effects of creep and shrinkage on the overall response of structures, such as the General Method[12] and the Age-Adjusted Effective Modulus Method.[13] These methods of analysis are complex and used in software, such as the General Method, which consists of combining a discretization in space and time using the recursive numerical solution of the Volterra integral of the Creep Constitutive Law. Three analysis methods with various degrees of simplification and accuracy applicable to most common structures with post-tensioned slabs are presented.

Precast Concrete Institute (PCI) Method

The PCI Method[8] is an analysis method (described in its Section 4.4.3, "Volume-Change Effects on Moment-Resisting Frame") for the effects of slab shortening in structures with prestressed slabs. Although it is limited in scope, it is based on sound principles. The PCI method provides tables to estimate slab shortening and simple equations to compute the buildup of restraint forces in slabs and beams, as well as the column forces and moments from an analysis with the equivalent frame simplification. Because it is based on the equivalent frame method, complex structural layouts are difficult to model—for instance, stiff walls and slabs with geometry irregularities and asymmetries. It does not account for Poisson's ratio or construction sequence. Relaxation of concrete is taken into account with fixed reduction factors applied indiscriminately to all kinds of concretes, environments, shapes, and any loading time and concrete age.

Commercial program integrated analysis (CPI)

Several commercial programs provide staged construction analyses that can be adapted to most designs, construction methods, schedules, and materials. Time stages are defined either by change in the structure geometry or loading and by aging of the structure. Staged construction is a type of nonlinear static analysis in which the structural material properties and geometry change during the analysis. The analyses apply to equivalent frames, 3-D frames, and 3-D finite element models. They provide displacements, shear forces, and moments in columns and walls, resultant stress in the slabs (including the stress changes from restraint forces) at any location and at any required time. Because all time-dependent functions are integrated in the program, it requires no theoretical knowledge from the user, or additional processing of input and output. These analyses are consistent with the General method presented in Reference 12. CPI analyses also provide the differential vertical shortening and could be compared with the method presented in Reference 16.

Temporary mitigation measures such as pour strips or temporary sliding supports are difficult to evaluate because in addition to the nonlinear properties of materials, the static scheme is also modified in time. CPI analysis may allow one to take into account both nonlinearities using the third and fourth theorems of linear viscoelasticity.[12]

Their main limitations are:

(a) Designers have no access to the algorithms used nor to the details and constants used in the creep and shrinkage models.

(b) Limited to one or two prediction models of creep and shrinkage.

(c) Staged construction analyses with time-dependent material properties are complex and time-consuming.

(d) There are several important modeling aspects of their application that are not mentioned or highlighted in the instruction manuals. For instance, it is critical to define stages with increasing time-steps. In the CPI analysis examples in this paper, the service life started after construction was finished, and time stages were set at 3 months, 1 year, 3 years, 10 years, and 30 years with each stage divided into 20 time-steps.

EQUIVALENT SHORTENING ANALYSIS (EQS)

This proposed analysis reduces the time-dependent analysis to an instantaneous analysis. It is more comprehensive and precise than the PCI Method, although it may not be as accurate and comprehensive as the CPI analysis. Nonetheless, it gives a clear understanding of the whole behavior of each step of the analysis and can be easily revised if needed. This method can use any creep and shrinkage model, including those calibrated with test results. The output provides deformations, forces, moments and stresses in columns, and walls and slabs at any desired age and location.

EQS analysis consists of two steps: and computation of the equivalent shortening, reduction factors, and equivalent elastic moduli with any creep model. In this paper, CEB M90 or B3 models were used.

Structural analysis to obtain restraint forces:

- Any FEM analysis software can be used. The unrestrained shortening is input as a strain to the slabs. Equivalent elastic moduli are used. Displacements, forces, moments, and stresses provided by the analysis are the desired relaxed results of the restraints.
- Alternatively, restraint forces can be computed with PCI Design Aids. In that case, the computed equivalent shortening is used.

Hypothesis

Equivalent shortening, reduction factors, and equivalent elastic moduli are defined in the Notation section and their calculation described in the flowchart in Fig. 7. Displacements and reduction factors can be computed for any of the models in ACI 209-2R[11] using the program Creep 2.0,[14] or Mathcad files in Reference 15. The main difficulty is to estimate the restraint force reduction due to the concrete relaxation—that is, the relaxation to a variable-imposed deformation $R(t,t_0)$, as shown in Fig. 4. The following hypotheses were made:

- The staged construction is considered in terms of the age of columns, walls, and slabs at post-tensioning.
- The steel prestress is considered in terms of an axial load applied along the perimeter of the slabs. Tendon locations and geometry are not considered.
- Prestress is assumed constant with time and equal to the initial prestress. Results are slightly conservative compared to those using the effective prestress.
- Slab shortening from elastic shortening, creep, shrinkage, and temperature (ES, CR, SH & T) are modeled as an axial uniform strain or as a uniform temperature drop.
- Vertical and horizontal deformations of walls and columns from ES, CR, SH & T are approximated as equal to those in the slabs.
- Shortening generates restraint forces, which in turn reduce the shortening with time. This analysis is a first iteration only.
- Creep models in ACI 209-2R,[11] calibrated or not, can be used, but the same model must be used in all calculations stages.

Compliance function and shrinkage strains

Equations (3) and (4) are from the B3 model and Eq. (5), and (6) are from CEB M90 model, as reported in ACI 209-2R[11] for the shrinkage strains and compliance function, respectively (refer to Reference 11 for notation).

$$\varepsilon_{sh.B3}(t)=\varepsilon_{sh\_\inf}(t_c)\cdot k_h\cdot sh_{B3}(t,t_c) \qquad (3)$$

$$J_{B3}(t,t_0,t_c)=q_1+C_0(t,t_0)+Cd(t,t_0,t_c) \qquad (4)$$

$$\varepsilon_{sh.CEB}(t)=\varepsilon_{cso}\cdot\beta_{s.t.tc}(t,t_c) \qquad (5)$$

$$J_{CEB}(t,t_0)=\frac{1}{E_{cmt.CEB}(t_0)}+\frac{\phi_{28.CEB}(t,t_0)}{E_{cm28.CEB}} \qquad (6)$$

The relaxation function is obtained solving the Volterra hereditary integral in Eq. (7). Equations (7) and (8) are equivalent, but Eq. (7) is more convenient for structural analysis programs. The integral is solved numerically14 for a constant unit strain using a discretized compliance function as shown in Eq. (9) and (10). The details of the calculation of $R(t,t_0)$ can

$$\varepsilon_{\sigma}(t)=\int_0^t J(t,t_0)d\sigma_{(t_0)} \qquad (7)$$

$$\sigma_{\varepsilon}(t)=\int_0^t R(t,t_0)d\varepsilon_{(t_0)} \qquad (8)$$

$$\Delta R_{(t_k.t_0)} = \left[\frac{\sum_{i-1}^{k-1}\left[J(t_k,t_i)+J(t_k,t_{i-1})-J(t_{k-1},t_i)-J(t_{k-1},t_{i-1})\cdot\Delta R_{(t_i,t_0)}\right]}{J(t_k,t_k)+J(t_k,t_{k-1})}\right] \qquad (9)$$

$$\Delta R_{(t_k,t_0)} = \sum_{i=1}^{k} \Delta R_{(t_i,t_0)} \tag{10}$$

Unrestrained slab shortenings

Shrinkage shortening is calculated with Eq. (11) using the shrinkage strain from prediction models.[11]

$$\Delta_{sh}(L,t) = \frac{L \cdot \varepsilon_{sh}(t)}{2} \tag{11}$$

The compliance provides the uniaxial elastic plus creep shortening as given by Eq. (12).

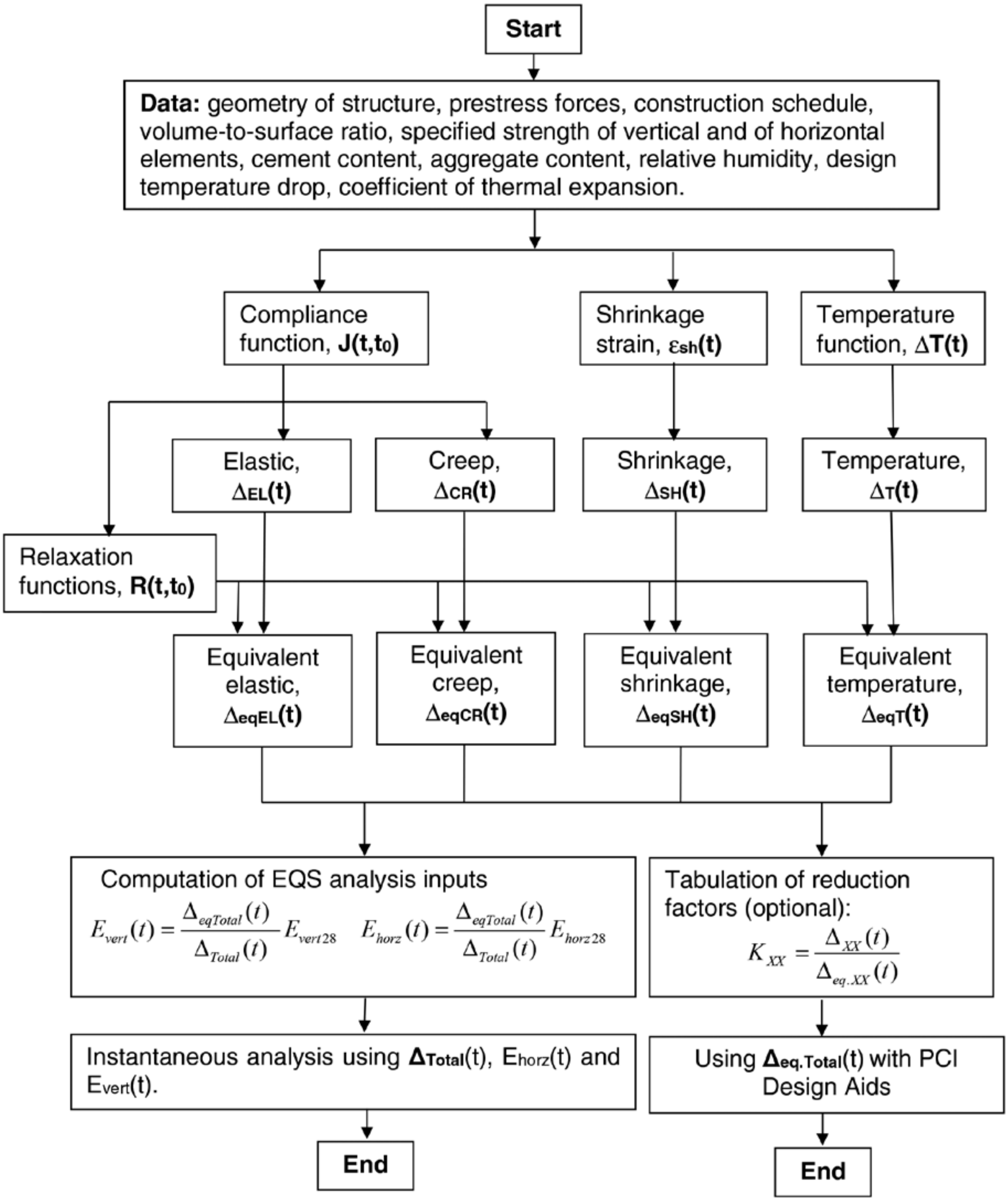

Fig. 7—EQS analysis flowchart.

$$\Delta_{el.cr.uniaxial}(PT,L,t) = -PT \cdot J(t,t_0) \cdot \frac{L}{2} \tag{12}$$

To obtain the creep shortening only, it is necessary to compute the elastic shortening and subtract it. For these shortenings, it is important to take into account Poisson's ratio (u) effect on the biaxial prestress, as in Eq. (13).

$$\begin{aligned}&\Delta_{el.cr}(PT_x, PT_y, L_x, L_y, t)\\ &= \Delta_{el.cr.uniaxial}(PT_x, L_x, t)\\ &- \frac{u \cdot L_x}{L_y} \Delta_{el.cr.uniaxial}(PT_y, L_y, t)\end{aligned} \tag{13}$$

Sudden temperature drops are conservatively considered without relaxation. Seasonal temperature drops are considered in Eq. (14) as a linear increase over the previous 6 months (180 days) before the evaluation (*evalt*).

$$\begin{aligned}&\Delta_{temp.seasonal}(L_x, \Delta T, t, evalt)\\ &= \left| \begin{array}{l} 0 \text{ if } t < evalt - 180 \\ \dfrac{-L_x \cdot \alpha_{temp} \cdot \Delta T}{2} \cdot \dfrac{(t - evalt + 180)}{180} \text{ if } evalt - 180 < t < evalt \end{array} \right.\end{aligned} \tag{14}$$

Equivalent shortening

Each displacement increment is reduced by the corresponding relaxation of the vertical elements (columns and walls). The displacement function starts at t_{const} to account for the age of columns and walls when the slabs are cast. The equivalent shortening is referenced to the elastic modulus of vertical elements at 28 days. Equation (15) is an application of Eq. (8) from Reference 12 with $S^{el,to}(t)$ taken as the shortening—that is, integrating over the age of the structure the relaxation function in Fig. 4 multiplied by each shortening increment of Fig. 1. Examples of $R(age,t)$ curves for a given age are in Fig. 4. The integral provides the equivalent shortening, $\Delta_{XXage.eq}$, at the desired age of the structure.

$$\Delta_{XX.age.eq} = \frac{1}{E_{cm28}} \int_{t_0 + t_{const}}^{age} R(age,t) \cdot \left(\frac{d}{dt} \Delta_{XX}(t - t_{const}) \right) dt \tag{15}$$

If restraint forces are computed with PCI Design Aids, this is the input needed.

Equivalent elastic moduli

The equivalent elastic modulus should be computed for each concrete mixture design used in the structure. At least, compute one for the vertical elements and one for the horizontal elements. Equation (16) shows how the vertical elastic modulus $E_{vert}(t)$ is reduced. The slab's $E_{horz}(t)$ can be similarly reduced, or since it has a lesser impact on the analysis, the relaxation in the slabs can be conservatively ignored.

$$E_{vert}(t) = \frac{\Delta_{eqTotal}(t)}{\Delta_{Total}(t)} E_{vert28} \tag{16}$$

COMPARISON OF PREDICTIONS

Comparison of results from analysis methods

The three methods are compared analyzing a four-story building with no mitigation measures (for example, pour strip, expansion joints, and sliding supports) because the slab is relatively short and the walls are favorably located. The main parameters of the structure, also shown in 3-D in Fig. 8 and with plan views in Fig. 11, are in Table 1.

Table 1: Main Parameters of Structure

Parameter	SI Units	in.-lb Units
Length in the x/y directions	48 m/36 m	157.5 ft/118.1 ft
Spans in the x/y directions	8 m/6 m	26.2 ft/19.7 ft
Slab thickness	160 mm	6.3 in.
Square columns	500 mm	19.7 in.
Wall thickness	250 mm	9.8 in.
Average ambient relative humidity	50% RH	50% RH
Design instantaneous temperature drop	-10 Δ°C	-18 Δ°F
Concrete specified strength at 28 days	34.5 MPa	5000 psi
Concrete specified strength at 3 days	20.7 MPa	3000 psi
Cement type	Type I portland cement	Type I portland cement
Poisson's ratio	$\nu = 0.18$	$\nu = 0.18$
Coefficient of thermal expansion	$\alpha_{temp} = 10^{-5}$ mm/(mm*Δ°C)	$\alpha_{temp} = (5.6^{*}10^{-6}$ in./(in.*Δ°F))

Tendons are banded in the long span direction and distributed in the other. The effective prestress (after prestress losses computation) in each direction is 1.044 MPa (151 psi), except at exterior spans with added tendons, where it is 1.965 MPa (285 psi). Reduction of prestress losses from slab shortening restraint is negligible. Columns and walls are considered fixed to the foundations. The Poisson's ratio was the same for elastic and for creep strains because the prestress in the slabs is lower than 5% of the mean strength at 28 days. The columns and walls are assumed to be 2 days old when slabs are cast. The slabs are post-tensioned at the third day after casting.

Shortening results

Figure 9 compares the unrestrained shortening in the x- direction for each method. CPI analysis uses the CEB M90 model and gives almost identical results as the EQS analysis using the CEB M90 model. EQS analyses with the B3 model use the same unrestrained slab shortening predictions, which are higher than the other methods. The PCI analysis predicts shortening between the CEB M90 and B3 results. When restrained, the slab shortening is reduced on the first floor while it is slightly increased on the second floor. The PCI method does not provide restrained slab shortening.

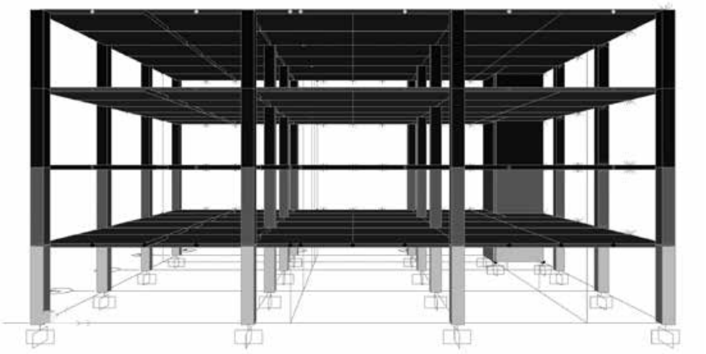

Fig. 8—View of one-quarter of structure, considering double symmetry.

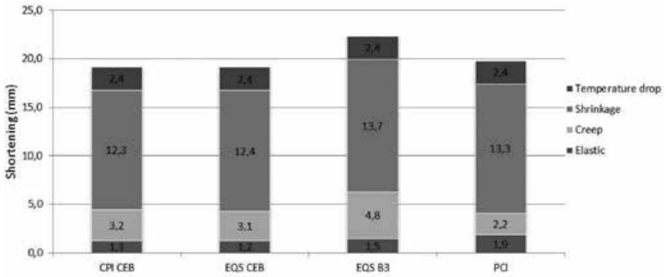

Fig. 9—Contribution of elastic, creep, shrinkage, and temperature drop shortenings to unrestrained slab shortening at 30 years. (Note: 1 mm = 0.039 in.)

Shear forces and moments in columns

Shear forces on an exterior column from the three analyses are compared in Fig. 10. Moment results are similar. CPI and EQS analyses based on the CEB model provide similar results, and are maximum after 30 years. EQS analyses using the B3 model provide similar forces and moments with a major difference: the maximum restraint forces occur at 2 years decreasing thereafter. The PCI method predicts much lower forces. All analyses show that the second floor is under additional compression, and that foundation restraint effect from columns and walls are negligible above the second floor. In this example, it is important to include shear forces and moments from slab shortening in column design.

Slabs affected by stress changes

CPI analysis results in Fig. 11 show the concrete resultant axial stresses considering restraint forces. Table 3 shows the results of each analysis in the center span of each floor. Concrete stress changes from all models are significant in the lower first two floors and negligible above these two floors. According to all analyses, the center spans of the first floor are the most critical ones.

Impact of foundation assumptions on slab stresses

Only walls located at the slab stiffness center can be ignored in shortening analyses (ignored in the PCI analysis). Concrete stress changes in the center spans depends highly

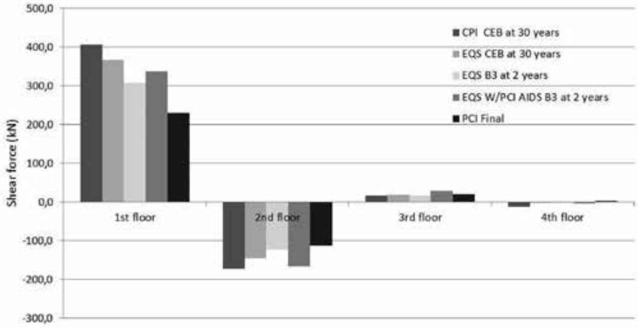

Fig. 10—Maximum shear forces at exterior column and age when they occur. (Note: 1 kN = 0.225 kip.)

on the hypothesis of columns fixed, pinned, or partially fixed to foundations. When considering the same structure but with pinned foundations, concrete stress reduction from restraint forces is only a one-third than with fixed foundations. For 50% partially fixed foundations, concrete stress reduction will be in between. In all models horizontal displacements at foundation level are prevented.

Changes in precompression stress over time

Figure 12 shows that the forces computed with the CEB M90 model are higher than those from the other models. EQS analyses based on the B3 model provide very different results with the forces increasing in the structure until their relaxation becomes more dominant than shortening; thereafter, forces decrease. Therefore, when using the B3 model, the long-term forces may not be the highest. In this example, the restraint forces reach their maximum at 2 years. This is a very important implication and should be linked to the conclusion on creep and shrinkage models in Reference 17. Albeit false, a common opinion of structural engineers is that most of the shrinkage occurs in the first few months. This assumption is somewhat justified by the results from the B3 model, because shortening rate is drastically reduced compared to the relaxation rate. Therefore, cracks from slab shortening appear either during the early years of the structure service life or never, despite

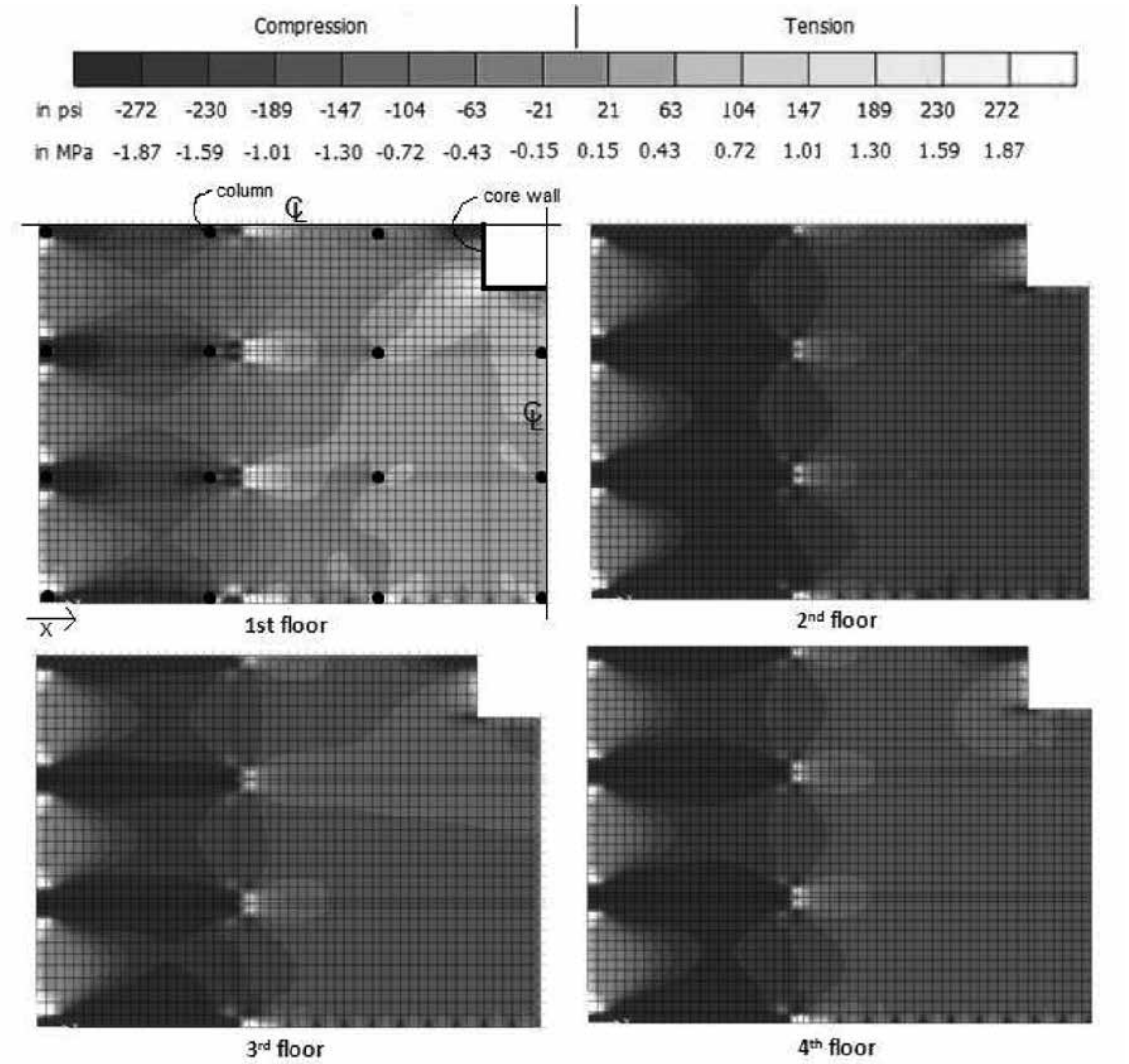

Fig. 11—CPI analysis concrete resultant axial stresses in long direction of four floors at 30 years. (Note: Higher prestress at exterior spans is due to additional tendons.)

progressing shortening with aging. PCI final values are the lowest ones because of the overestimation of the relaxation of forces implicit in the reduction coefficients provided. Additionally, they are not appropriate for early-age estimations because relaxation is considered constant; independent of the age of concrete and of the structure; and because the elastic displacements are neglected, although they are very important at early ages.

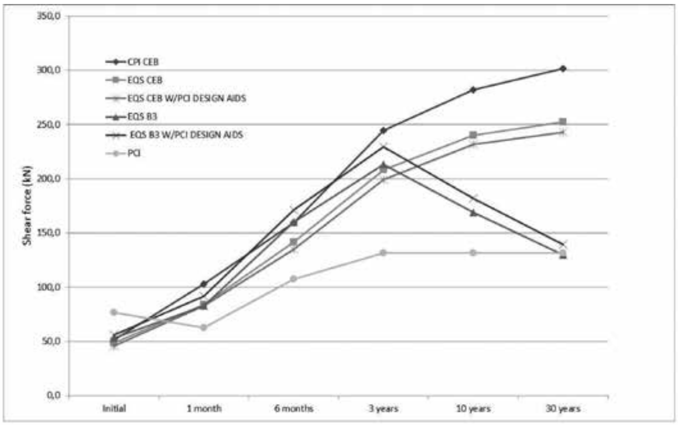

Fig. 12—Shear force from elastic, creep, and shrinkage shortenings at first-floor exterior column over time. (Note: 1 kN = 0.225 kip.)

DISCUSSION

Models for creep, shrinkage, and relaxation of concrete

Models for creep, shrinkage, and relaxation of concrete provide significantly different results for the same inputs, particularly their differences in their time rates. This is obvious from relaxation functions shown in Fig. 3 and 4.

Role of concrete relaxation

Relaxation of forces and moments is ignored in some recent research papers,[3,5] while it is roughly estimated by the PCI method[8]; refer to Table 2 for some reduction factors computed with EQS analyses. These references considered cracked section properties. However, reliefs from cracked sections and from concrete relaxation are distinct aspects of the problem and may be cumulative. Also, it is sometimes believed that forces in the structure are proportional to the displacements, which is not the case because of the concrete relaxation.

Relative importance of each source of slab shortening

The relative importance of each source can vary widely from one structure to another. For example, an open parking structure in an extreme cold weather region will experience far more slab shortening than in a climate-controlled building. From the configurations studied, it

Table 2—Example of reduction factors

f_c' = 34.5 MPa (5000 psi), slab depth = 16 cm (6.3 in.), and post-tensioning on third day								
	50% RH				80% RH			
Age	K_{EL}	K_{CR}	K_{SH}	K_T	K_{EL}	K_{CR}	K_{SH}	K_T
Using the B3 model								
1 month	3.08	2.91	2.20	1.00[*]	2.95	2.82	2.04	1.00[*]
2 months	3.81	3.04	2.06	1.00[*]	3.56	2.93	1.93	1.00[*]
1 year	7.17	3.59	2.95	1.26[†]	5.91	3.25	1.88	1.04[†]
10 years	16.36	4.65	3.86	0.80[†‡]	11.51	3.69	2.36	0.72[†‡]
30 years	20.64	4.92	4.97	0.70[†‡]	15.77	3.88	3.17	0.69[†‡]
Using the CEB M90 model								
1 month	2.64	2.46	2.24	1.00[*]	2.16	2.07	1.83	1.00[*]
2 months	3.23	2.62	2.27	1.00[*]	2.54	2.15	1.89	1.00[*]
1 year	5.58	3.15	2.52	1.71[†]	4.00	2.49	2.06	1.47[†]
10 years	9.17	3.52	2.58	1.33[†]	6.19	2.77	2.14	1.19[†]
30 years	10.24	3.53	2.54	1.26[†]	6.17	2.83	2.12	1.15[†]

[*] For daily or weekly temperature drops, use 1.0.
[†] These factors apply to seasonal temperature drops only.
[‡] Reduction factors are less than 1.0 because there is very little relaxation and the elastic modulus at loading is much higher than the 28-day modulus.

appears that shrinkage is the most important source of shortening and of restraint forces. Temperature drops may be critical when combined with shrinkage in an open structure. Restraint forces from post-tensioning (elastic and creep) are minor in the long-term because they are almost totally relaxed. However, restraint forces from elastic shortening are important when the structure is very young.

Rigidity of foundations

Assuming fixed connections of columns and walls to foundations may be too conservative when studying slab shortening. On the other hand, pinned connections are nonconservative. Further research is needed to develop comprehensive details and methods that will provide the rotational and displacement stiffnesses of foundations and their variations with time, based on the soil-structure interaction for most common types of soils and foundations.

Commercial program-integrated analysis

Commercial program-integrated (CPI) analyses are elaborated, complex, and comprehensive; but they may require some time to master. The main advantage of the CPI analysis is that it can be adapted to a wide range of post-tensioned concrete structures with almost any kind of structure geometry, construction sequence, post-tensioning systems, or materials. Also, the CPI analysis doesn't require an advanced knowledge of creep, shrinkage, and relaxation prediction models. It is sufficient to understand the mechanisms involved in slab shortening analyses because all the calculations of these functions are integrated in the program. For the same reason, it is more difficult to check the calculations and it is limited in terms of the available creep and shrinkage models used. With additional work, the CPI analysis can be used to compute deflections over time and differential vertical shortening of the structure.

Table 3—Comparison of resultant stresses at center span

Resultant stress at center span when restraint forces are maximum in MPa (psi)*					
Method model, age	CPE CEB M90, 30 years	EQS CEB M90, 30 years	EQS B3, 2 years†	EQS B3 with PCI design aids, 2 years†	PCI PCI tables, final
First floor	0.367 (53)	0.383 (56)	0.083 (12)	0.138 (20)	–0.321 (–47)
Second floor	–1.519 (–220)	–1.489 (–216)	–1.379 (–200)	–1.487 (–216)	–1.310 (–190)
Third floor	–0.932 (–135)	–0.976 (–142)	–1.000 (–145)	–0.957 (–139)	–0.967 (–143)
Fourth floor	–1.037 (–150)	–1.038 (–151)	–1.038 (–151)	–1.038 (–151)	–1.038 (–151)

*Positive sign is tension.

†The B3 model predicts maximum restraint forces at 2 years.

Table 4—Recommended reduction factors and results for examples

Recommended by	**Age at maximum restraint forces**	**Reduction factors**			
		K_{EL}	K_{CR}	K_{SH}	K_T
PCI seventh edition[8]	Final*	N/A†	4 to 6	4 to 6	1.5
Klein and Linderberg[9]	Final*	N/A†	2 to 3	2 to 3	1.5
Computed results range for examples					
B3 Model	Variable‡	6 to 8	3 to 4	1.3 to 3	1 to 1.5
CEB M90 Model	30 years	6 to 11	2.5 to 4	2 to 3	1 to 1.5

*"Final" is not defined (next-to-last value being 5 years).

†Although neglected for precast concrete structure (occurring in the plant), must be accounted for in P/T slabs.

‡With B3 model, maximum restraint forces occur earlier decreasing after peak. Conservatively, this peak could be taken at 3 years.

Equivalent elastic moduli analysis

Equivalent elastic moduli (EQS) analyses presented in this paper reduce the problem to instantaneous analyses using equivalent elastic moduli. EQS analyses provide both displacements and forces to be used for design. A programming tool such as Mathcad or Mathlab is needed for the preprocessing of the elastic moduli, but the instantaneous analysis can be done with a basic structural program. EQS results can be used with PCI Design Aids to compute restraint forces. The processing can be adapted to any creep and shrinkage model, as shown in Reference 15.

Updating PCI tabulated strains and reduction factors

Comparisons in Table 4 show that PCI analysis reduction factors are outdated. Tables similar to Table 2 could be developed for most common applications. Not only do reduction factors vary widely from one concrete to another but also vary between models. Additionally, maximum restraint forces may not occur at the same time.

CONCLUSIONS

Even though it is not usually taken into account through analysis in the design, the designer should consider the following concepts:

- Concrete relaxation must be considered in the analysis of the structural effects of post-tensioned slab shortening.
- Shrinkage and temperature drops are the two main sources of forces and moments from shortening of post-tensioned slabs.
- Creep and shrinkage models in ACI 209.2R-08[11] produce different results and behaviors in the structural analysis. In the CEB M90 model, the most critical restraint forces and moments occur at late ages and continue to increase with time. For the B3 model, restraint forces are maxima in the first years before decreasing because of the difference between the shrinkage and relaxation rates, while strains continue to increase. This behavior predicted by the B3 model is confirmed by the observation of cracking and deformation in structures over time but it needs further research.[17]
- There is a need for a detailed and comprehensive method to determine the time-dependent rigidity of foundations because the restraint from foundations is the most critical factor influencing the effects of slab shortenings.
- Three analysis methods were discussed from the simpler to the more comprehensive: PCI, EQS, and CPI. Their results allow the evaluation of restraint forces and if mitigation measures are needed.

NOTATION

CR = time-dependent volume-change of slab under sustained post-tensioning forces (creep)

ES = instantaneous volume-change of slab from post-tensioning forces (elastic)

E_c = column modulus of elasticity

$E_{cm28.CEB}$ and $E_{cm.CEB}(t)$ = mean elastic moduli according to CEBM90 model at 28 days and at age t, respectively

$E_{horz}(t)$ and $E_{vert}(t)$ = equivalent elastic moduli of horizontal and vertical elements, respectively, accounting for reduction of forces and moments in structure from concrete relaxation at age t of structure

h_s = story height

I_{col} = column uncracked inertia (service limit state)

$J(t,t_0)$ = compliance function is total load induced strain (elastic plus creep) at age t per unit stress caused by unit uniaxial sustained load applied at time t_0

k_F and k_m = coefficients given for each floor by Fig. 4.11.24[8] depending on following factors: number of stories, stiffness ratios of horizontal/vertical elements, and type of base fixity (fixed or pinned)

$K_{XX}(t)$ = reduction coefficient for shortening from *XX* (standing for EL, CR, SH, T or Total to account for concrete relaxation of forces at given time

$R(t,t_0)$ = relaxation function is stress response at time t to a sustained constant unit strain applied at time t0

SH = time-dependent volume-change caused by concrete shrinkage

T = time-dependent volume-change caused by short term or seasonal temperature drops

$\Delta_{XX}(t)$ = unrestrained shortening (displacement) from *XX* (standing for EL, CR, SH, T or Total) at a given time t

$\Delta_{XX.Eq}(t)$ = equivalent shortening from *XX* (standing for EL, CR, SH, T or Total). Applying corresponding strain in instantaneous analysis (with E_{cm28}) provides restraint forces. It also can be used directly in PCI method

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