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SHEAR DESIGN OF POST-TENSIONED CONCRETE DIAPHRAGMS—COMPARISON OF TWO DIFFERENT APPROACHES

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SHEAR DESIGN OF POST-TENSIONED CONCRETE DIAPHRAGMS—COMPARISON OF TWO DIFFERENT APPROACHES

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Most concrete structural elements have multiple design examples that can be located in textbooks, magazine articles, or found on the internet. One element that is lacking in examples and explanation is the design of concrete diaphragms, which is arguably the most important part of the lateral system. Regardless of how strong, ductile, and well detailed your shear wall or moment frame is, if the diaphragm is unable to deliver the load to these elements, their design is irrelevant. Unfortunately, the information that is available for concrete diaphragms typically requires the engineer to use a wood or steel design philosophy. While the wood method can be used, it effectively ignores the inherent shear strength of concrete and replaces that strength with a significant amount of nonprestressed reinforcement. The following example will demonstrate the wood diaphragm methodology and compare that to a design that is based on standard concrete principles that are already used for a wide variety of concrete members.

With the addition of the specific diaphragm chapter in ACI 318-14[1] (Chapter 12) it is critical that engineers are allowed to have the choice to use basic concrete principles rather than be handcuffed to a procedure that was developed for a different material. The concrete-specific diaphragm design that is presented is not new. This approach has been successfully used by post-tensioned (PT) concrete engineers for decades and has been incorporated into hundreds of millions of square feet of construction.

The building shown in plan and elevation in Fig. 1 will be the basis for the example design of the roof level diaphragm. The structure is an 8 in. (203 mm) post-tensioned flat plate supported by reinforced concrete columns and 14 in. (355 mm) thick shear walls. The slabs are assumed to have a precompression value of 175 psi (1.2 MPa). Each level will have a 10 ft (3.0 m) floor height. The slabs will use a 28-day concrete compressive strength of 5000 psi (34 MPa), unbonded tendons with Grade 270 (1860 MPa), 0.5 in. (13 mm), 7-wire, low relaxation strand and Grade 60 (410 MPa) nonprestressed reinforcement . Each floor has a seismic area dead load of 160 psf (7.7 kPa) which generates a total weight of 5184 kip (23,000 kN) per level. For simplicity, the weight of the roof and the floors are assumed to be equal. The structure is designed with a base shear of 4329 kip (19,000 kN). Based on the vertical distribution per ASCE 7[2], Section 12.8.3, the roof level will have a lateral and diaphragm force of 1443 kip (6400 kN).

The shear and moment diagram shown in Fig. 2 is generated by using a rigid diaphragm model with the lateral load being applied in the North/South direction. All shear walls are 14 in. (355 mm) thick with a compressive strength of 5000 psi (34 MPa). Due to the symmetric nature of the building and wall layout, no torsion is created so the walls in the East/West (Grid A and B) direction do not resist any lateral forces. For diaphragm design, the accidental torsion per ASCE 7 cannot be applied because that is a fictious value and creates an offset between the center of mass and the center of applied loading. If the shear wall reactions are used from a model that includes accidental torsion, the moment diagram will not close due to this discrepancy. For this reason, a separate analysis must be performed to achieve a statically correct shear and moment diagram.

Design Collector on Grid 1 Using Wood Diaphragm Analogy

Figure 3 is an example taken from Donald E. Breyer's[3] book ***Design of Wood Structures – 3rd Edition***.

In the wood diaphragm analogy, the collector force is determined by first dividing the wall reaction by the full length of the diaphragm along that line. From Fig. 3, each side of Grid 2 will resist a lateral shear load. The Grid 1 side resists 10.5 kip (300*(70'/2)') [47 kN]

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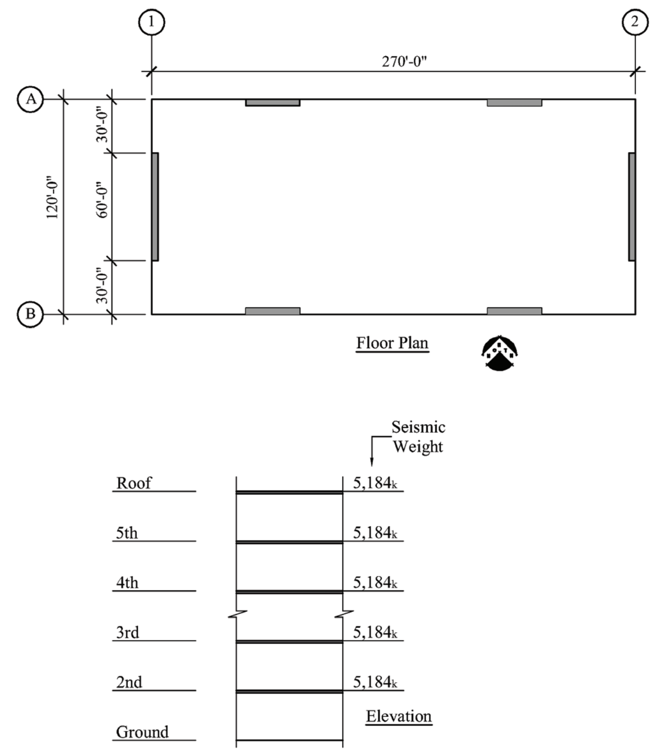

Fig. 1—Example building plan view and elevation. (Note: 1 kip = 4.45 kN.)

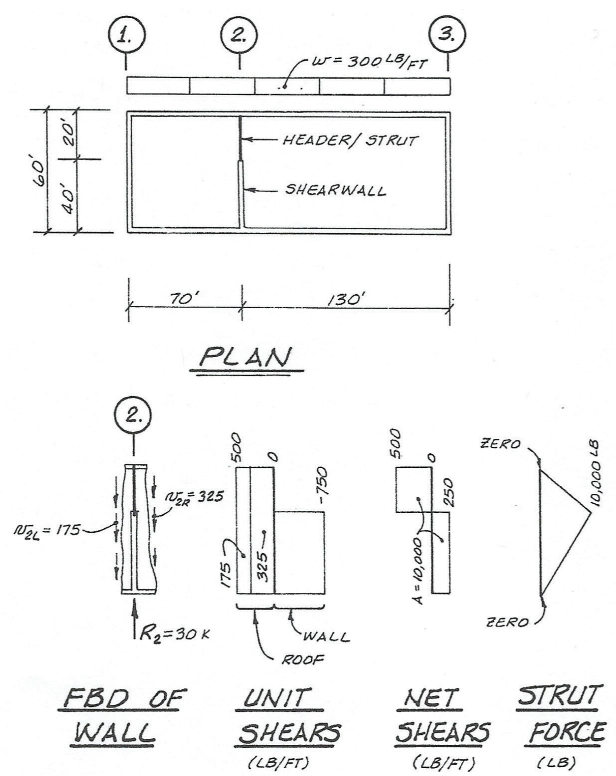

Fig. 3—Wood diaphragm design example. (Note: 1 k = 4.45 kN; 1 ft = 0.3 m.)

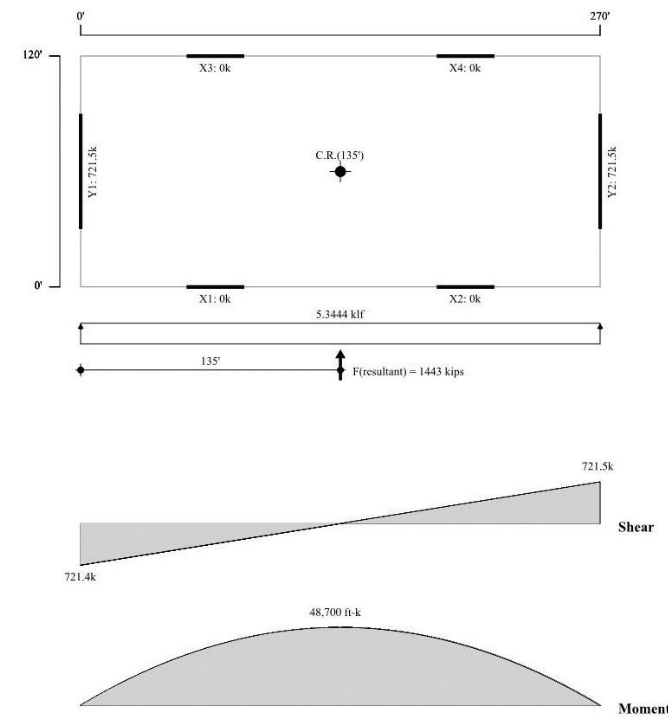

Fig. 2—Statically correct shear and moment diagram – north/south loading. (Note: 1 k = 4.45 kN; 1 ft = 0.3 m.)

while the Grid 3 side resists 19.5 kip (300*(130'/2)) [87 kN]. The force on each side divides their force by the total length of 60 ft (18 m) creating force of 175 and 325 lb/ft (2.6 and 4.7 kPa/ft) over the length of the diaphragm. Provided the plywood diaphragm shear strength is larger than each of these values, the diaphragm can successfully deliver the load to the collector line. Combining the two sides creates a total shear load of 500 lb/ft (7.3 kN/m). Without going into a discussion on the ASCE 7[2] over strength (Ω_o) factor, the statically correct collector force is the amount of load between the edge of the diaphragm that is "dragged" to the shear wall. The main premise of the wood analogy is that any portion of the diaphragm not directly connected to the shear wall must be collected and delivered to the shear wall. Per Fig. 3, the collector force is 500 lb/ft (7.3 kN/m) multiplied by 20 ft (6.1 m), which creates a collector force of 10,000 lbs (44 kN). Note in this method, if the diaphragm shear strength is substantially stronger than required, there is no change in the collector force. If the diaphragm can resist 1 kip/ft (14.6 kN/m) or 10 kip/ft (146 kN/m), the collector design is the same.

Using the same wood collector analogy for our concrete building, the Grid 1 shear wall reaction of

721.5 kip (3200 kN) is divided by the depth of the diaphragm to create a unit diaphragm force.

$$V_{COLLECTOR} = 721.5\,\text{kip} / 120\,\text{ft} = 6.0\frac{\text{kip}}{\text{ft}}\left(87.6\frac{\text{kN}}{\text{m}}\right)$$

The engineer would now check this demand against the shear capacity of the diaphragm. The exact diaphragm shear strength will be calculated in the next section. At this point, let us assume that the capacity of the diaphragm exceeds the applied unit force of 6.0 kip/ft (87.6 kN/m).

The unit shear in the wall is

$$V_{WALL} = 721.5\,\text{kip} / 60\,\text{ft} = 12.0\frac{\text{kip}}{\text{ft}}\left(176\frac{\text{kN}}{\text{m}}\right)$$

Per the loading shown on Fig. 4, the static collector force at each end of the wall is 180 kip (30 feet × 6.0 kip per foot) [800 kN]. The design connection force as required by ASCE 7-16[2], Section 12.4.3.1 (Eqn. 12.4-7) is

$$E_{mh} = \Omega_o \times Q_E$$

Where $\Omega_o = 2.5$ (ASCE 7-16[2] Table 12.2-1,) – Building Frame System with Special Concrete Shear Walls

$$\therefore F_{COLLECTOR} = 2.5 \times 180\,\text{kip} = 450\,\text{kip}\ (2000\,\text{kN})$$

In the wood analogy, the collector is an element in-line with the vertical seismic resisting element (the shear wall). In most typical wood structures, the collector would be a beam that directly connects to the shear wall (Fig. 5). The same concept is used in steel framing.

ACI 318-19[4], Section 12.5.1.4 states that, "it shall be permitted to use precompression from prestressed reinforcement to resist diaphragm forces." However, if the wood analogy is used, the only collector element is the portion of the diaphragm that is directly in-line with the concrete shear wall.

Therefore, the only consistent application of this ACI section (in the author's opinion) would be to apply the residual precompression over the localized collector area that is in-line with the shear wall. For the flat plate structure, that area is the thickness of the slab multiplied by the width of the wall. Typically, the residual post-tensioning available for seismic diaphragm design is approximately 20% of the total precompression. Without going through the analysis to determine the residual force of a two-way post-tensioned flat plate, the amount of precompression that can be used to reduce the collector force assuming the 20% value is

$$F_{Preccompression} = 0.20 \times (0.175\,\text{ksi} \times 8\,\text{in.}\ \textit{slab} \times 14\,\text{in.}\ \textit{shearwall}) = 3.9\,\text{kip}\ (17\,\text{kN})$$

The 175 psi (1.21 MPa) re-compression value is listed in the description of the building. While this small

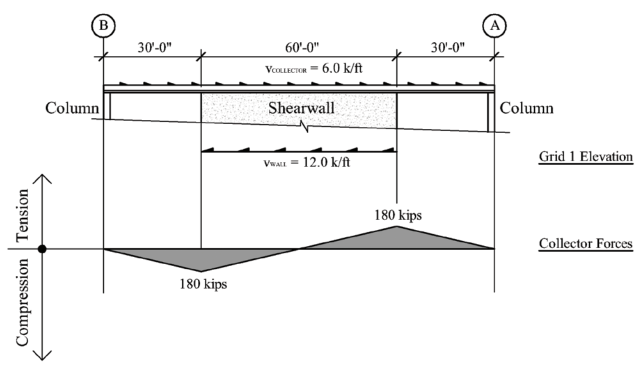

Fig. 4—Collector force along Grid 1 wall.

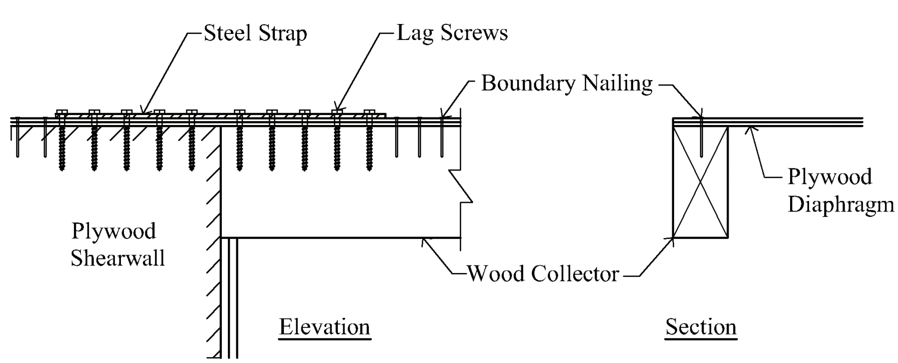

Fig. 5—Typical wood collector to shear wall connection.

precompression force could be used to decrease the 450 kip (2000 kN) collector force, it hardly seems worthwhile. For the remainder of the wood method example, the 3.9 kip (17 kN) of residual precompression will be ignored.

Therefore, the collector reinforcement required using the full collector force of 450 kip (2000 kN) is

$$A_{s\_Required} = \frac{F_{COLLECTOR}}{\phi F_y} = \frac{450\,\text{kip}}{(0.9)(60\,\text{ksi})} =$$

$$8.33\,\text{in.}^2\,(5370\,\text{mm}^2)$$

$\therefore$ Use (14)-#7 Collector Reinforcing Bars

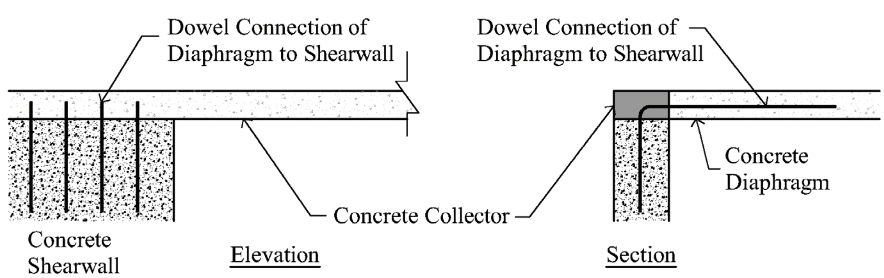

Fig. 6—Typical concrete collector to shear wall connection.

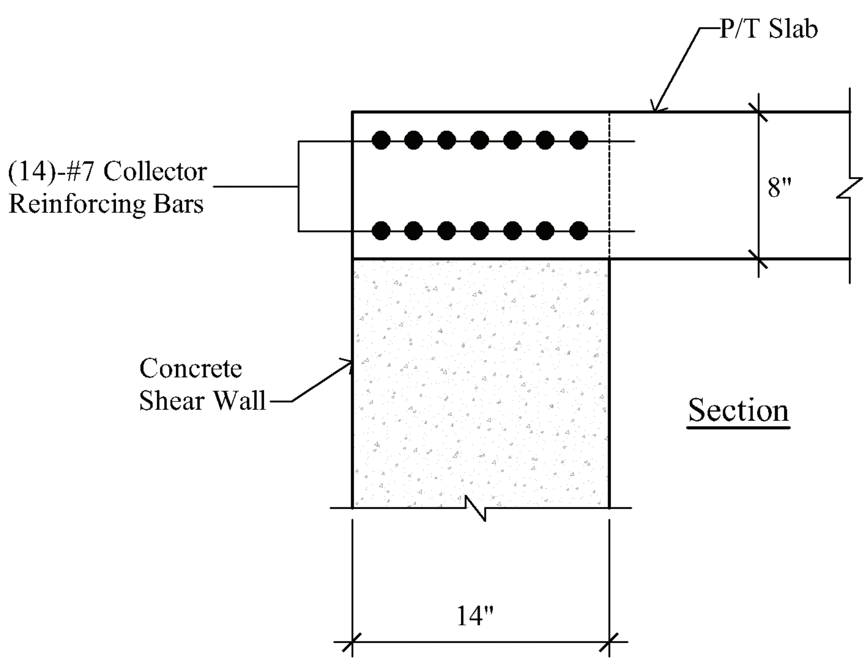

Fig. 7—Collector Bars in slab.

In the wood methodology, a tremendous amount of reinforcing must be placed in the relatively thin 8 in. (203 mm) slab as shown in Fig. 7. Even if it's believed that the collector nonprestressed reinforcement does not have to be in line with the wall, the 14 No. 7 bars are still required and add a significant amount of nonprestressed reinforcement (which will impact the cost) to the system. When combined with the other reinforcing (post-tensioning, distributed bottom reinforcing, top flexural reinforcement, dowels, and so on) the only realistic way to achieve well consolidated concrete is to locate the collector bars in a beam. In addition to the expense, beams may require an increased floor height and may have an impact on the architectural elements of the structure. The two-way flat plate structure that has been constructed for decades, suddenly has challenges to satisfy code without a noticeable amount of additional nonprestressed reinforcement. The other concern with using the wood methodology is there is no reduction in collector nonprestressed reinforcement based on the diaphragm strength. If the diaphragm has a capacity of 10, 20, or 100 kip/ft (146, 292, or 1460 kN/m), the same 14 No. 7 bars are required. Once the diaphragm is shown that it can transfer the lateral load, any additional strength provides no benefit. This is in stark contrast to other reinforced concrete elements. Higher strength concrete in columns, beams, slabs, and shear walls require less nonprestressed reinforcement for the same strength. Fortunately, there is another more appropriate way to design concrete diaphragms.

To keep the focus on the difference between the two analysis methods, the example is not going to cover the compression portion of the design per ACI 318-19,[4] Section 18.12.7.5. This section will require transverse reinforcement if the compressive stress exceeds $0.2f_c'$. If a collector beam is used or the compression force is limited to a certain slab width, the section may require additional detailing. Based on the interpretation of the code used in this example, the compressive stress in the slab over the shear wall will easily exceed $0.2f_c'$ Because the detailing requirements are not specific to post-tensioning, a discussion on this section will be deferred to another article.

Grid 1 Diaphragm Design Using Accepted Concrete Principles

When the floor system is designed acknowledging that the diaphragm is effectively a concrete beam, the load path and shear strength can be used to eliminate or reduce any collector reinforcement. Similar to a concrete beam, increased compressive strength, residual precompression, and nonprestressed reinforcement can be used to strengthen the system to help in load transfer.

Per the loading diagram in Fig. 8, the diaphragm is modeled as concrete beam supported at each end by the Grids 1 and 2 shear walls. As in a traditional concrete beam, the shear failure plane emanates from the end of the support at 45 degrees, and the shear reinforcement (collector reinforcement) only needs to cross the shear failure plane to be useable at that location. This is very different than in the wood method where all the collector reinforcement must occur in line (or very close) with the seismic element (shear wall).

The 45-degree shear plane (load path) is based on ACI 318-19, Section 9.4.3.2, figure R9.4.3.2a (Fig. 9) and has been used in the design of the beams for decades. Most engineers will assume Fig. 9 is an elevation where the vertical support is a column and the horizontal member is a beam. However, if this model is rotated so the figure is viewed in plan, the columns are now the shear walls and the beam is the diaphragm. In this configuration, does anything conceptually change? How will the concrete know to perform in a certain way as a beam, but do something different as a diaphragm?

Per Fig. 8, the shaded compression zone has a direct load path to the support. Similar to concrete beams, this area can be subtracted from the total shear wall reaction since it does not rely upon the shear strength of the diaphragm to be transferred. This is allowed per section 9.4.3.2 of ACI 318-19[4] and has been a common approach in the design of beams, girders and footings. The one difference is the diaphragm load is generated by the mass of the structure so only the compression area can be subtracted. While Fig. 8 loading is shown in the more traditional beam approach, it needs to be understood that it is a representation to generate shear and moment diagrams. In this example, the diaphragm area force in the direct compression zone is calculated by

$$Area = {(90 \times 90\,\text{ft})}\big/{2} = 4050\,\text{ft}^2\,(376\,\text{m}^2)$$

$$\text{Direct compression area force} = 4050\,\text{ft}^2\left(\frac{1443k}{120 \times 270\,\text{ft}}\right) =$$

$$180.4\text{ kip } (802.5\text{ kN}$$

The loading that is required to be transferred by shear is 721.5 k – 180.4 k = 541.1 kip (2407 kN)

The direct compression force needs to be checked in each direction. Because the building is symmetric and the shear walls are centrally located, the direct compression zone is the same in both loading directions. Without doing anything besides using code accepted concrete principles, the concrete diaphragm has reduced its shear demand by 25%. While the shear demand has gone down, the shear friction dowels required to transfer the load into the shear wall are not affected by the compression zone. Regardless of the method used, the connection across the construction joint between the slab and shear wall must account for all of the load.

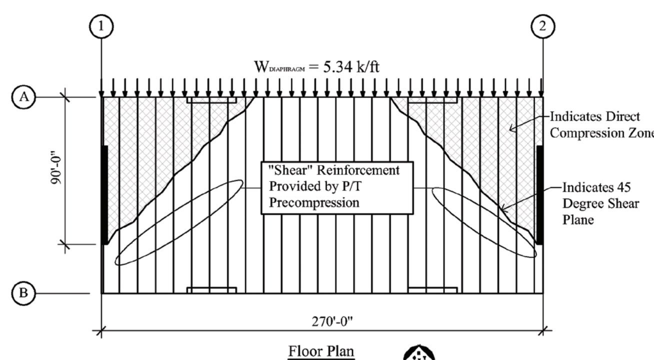

Fig. 8—Diaphragm loading. (Note: 1 kip = 4.45 kN; 1 ft = 0.3048 m.)

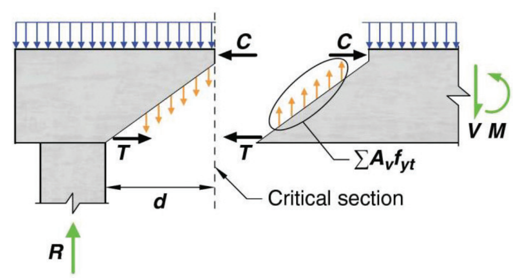

***Fig. R9.4.3.2a**—Free body diagrams of the end of a beam.*

Fig. 9—Critical section per ACI 318-19, Section 9.4.3.2.

A more impressive benefit of the direct compression zone can be seen when we analyze the walls on Grids A and B. With the diaphragm load in the west direction, the 45 degree shear planes in Fig. 10 emanate off the ends of the four walls. Due to wall layout, most of the diaphragm load will go directly to the walls without using post-tensioning, nonprestressed reinforcement, or concrete shear strength. This is simply due to the nature of concrete that we have used in beam design for decades. The only portion of the diaphragm that requires shear strength is the area outside of the hatch. In Fig. 10, the post-tensioning is shown as banded tendon groups to match what is typically designed. There is no difference in the shear strength or analysis method if the tendons are in the uniform or banded direction.

The wood methodology would ignore the compression zone and replace this with a line of nonprestressed reinforcement along Grids A and B for the length of the building. While doing this probably will not have a negative impact structurally, the collector reinforcement will add cost to the structure that may not be required. With a large or multi story structure, this could add up to a noticeable amount of additional costs.

Determining Useable Precompression in Diaphragm

There are various ways to determine the amount of residual post-tensioning precompression available for use in the seismic diaphragm. The typical process consists of acknowledging that in a catastrophic seismic event the service level requirements of post-tensioned concrete no longer apply. The only requirement is that the slab maintain the strength to support the vertical loads while also acting as a diaphragm to distribute the seismic inertial loads to the lateral force resisting elements. Most engineers will re-analyze their slabs using the appropriate load case for dead, live, and seismic loads and determine the minimum amount of post-tensioning required to satisfy ultimate strength. The load cases per ACI 318-19,[4]

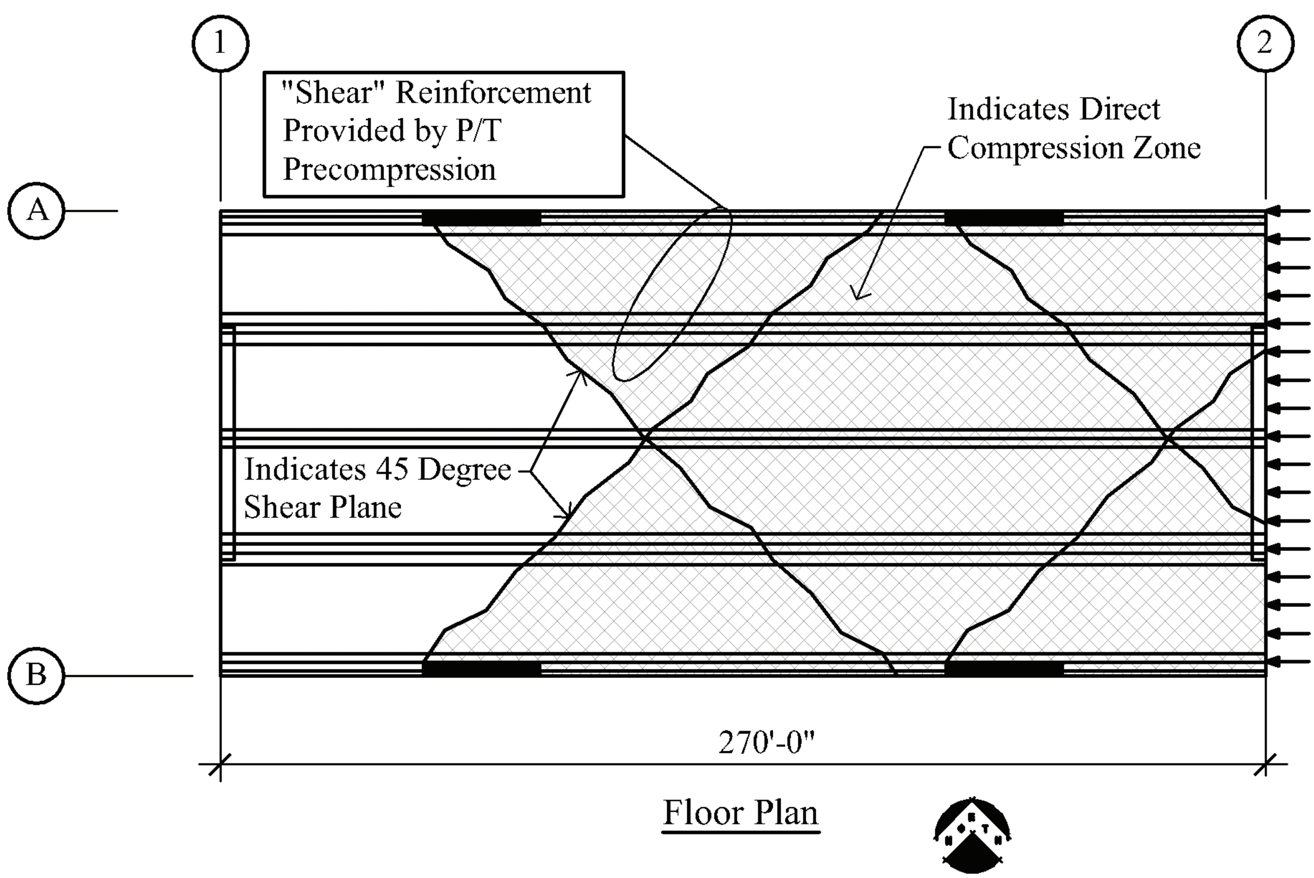

Fig. 10—Diaphragm loading in west direction.

Section 5.3 or ASCE 7-16[2] chapter 2 should be used at the engineer's discretion.

For example, the typical ultimate strength design is $1.2DL + 1.6LL + 1.0M_2$ per ACI 318-19[4] Equation 5.3.1b. Equation 5.3.1c has $1.2DL + 1.0LL + 1.0E + 1.0M_2$. Assuming there is no E component for the slab design (E is resisted by the shear walls), the difference in the pre-compression will be due to the change in live load (LL) load factor and the secondary moment due to the reduction in post-tensioning. The amount of nonprestressed reinforcement on the drawings is a constant and only the post-tensioning force is adjusted. The drape of the tendons cannot be changed in this analysis. In some cases, the amount of post-tensioning required to satisfy ultimate strength may be below the 125 psi (0.86 MPa) code minimum. In the author's opinion, because stresses and serviceability are not a consideration during a seismic event, there is no minimum precompression requirement. The difference in post-tensioning between what is shown on the drawings and what is required by the second (ultimate strength only) analysis is the amount of precompression available for use in the diaphragm. The pre-compression is typically viewed as a service level value, however designers do not increase it by the typical 1.4 multiplier to transform service to ultimate strength. The residual pre-compression calculated as described is directly used in the diaphragm shear equation.

The book "Post-Tensioned Concrete Principles and Practice - 4th Edition[5]" goes through a more rigorous analysis, demonstrating exactly how this analysis is done. However, for most "normal" designs, the ultimate strength analysis results in approximately 20% of the total precompression in the slab being available for use in the seismic diaphragm.

For this example, we will conclude that 3.5 kip/ft (51 kN/m) of residual precompression can be used in the seismic design.

Calculating Diaphragm Shear Capacity

ACI 318-19,[4] Section 18.12.9.1 and 12.5.3.3 states that the diaphragm shear capacity is to be calculated as follows

$$V_n = A_{cv}\left(2\sqrt{f_c'} + \rho_t f_y\right) \leq 8A_{cv}\sqrt{f_c'}$$

In the post-tensioned concrete system, the residual precompression can be added to the mild reinforcing part of this equation. The code does not allow the engineer to use a yield value of 270 ksi (1860 MPa) for the post-tensioning. Only the precompression can be used per ACI 318-19,[4] Section 12.5.1.4 and 18.12.7.2. The f_y value in the equation only applies to nonprestressed reinforcement. For this example, we conservatively ignoring any additional strength from nonprestressed reinforcement.

$$\therefore V_n = (8\text{ in.})(12\text{ in.})\left(2\sqrt{5000}/1000\right) + 3.5\text{k}/\text{ft} = 17.1\text{k}/\text{ft}\ (250\text{ kN/m})$$

$$V_{n\_\max} = 8 \times (8\text{ in.})(12\text{ in.})\sqrt{5000}/1000 = 54.3\text{ k/ft }(792\text{ kN/m}) >> 17.1\text{ k/ft }(250\text{ kN/m})$$

The useable shear capacity is

$$\phi V_n = 0.60\ (17.1\text{ k/ft}) = 10.26\text{ k/ft }(150\text{ kN/m})$$

(Note that this is much larger than the 6.00 k/ft (88 kN/m) required by the wood analogy method)

Per the previous analysis, the total shear diaphragm (collector) demand is 541.1 kip (2400 kN). This value was due to the reduction of the direct compression area.

The shear (collector) capacity of the diaphragm shown in Fig. 8 is calculated as follows

$$\phi V_n = 10.26\text{ k/ft} \times 90\text{ ft} = 923.4\text{ kip }(4100\text{ kN}) >> 541.1\text{ kip }(2400\text{ kN})$$

$\therefore$ No additional reinforcement is required

The 90-foot dimension is the depth of the beam created off the end of the shear wall, which acts at the diaphragm support. In designing the concrete diaphragm like a concrete beam, this example illustrates the engineer can eliminate the need for additional collector reinforcement required by the wood methodology. The lack of collector reinforcement would apply to all the shear walls on the project and will hopefully result in a noticeable reduction in nonprestressed reinforcement, congestion and associated costs.

To further demonstrate the capacity of a typical concrete diaphragm, let us reconsider our example by **ignoring** the direct compression zone and residual precompression and just use the concrete. The code concrete diaphragm shear strength is

$$\phi v_c = 0.60\ (8\text{ in.})(12\text{ in.})\left(2\sqrt{5000}/1000\right) = 8.15\text{ k/ft }(120\text{ kN/m})$$

The shear (collector) capacity of the concrete only portion of the diaphragm in Fig. 8 is calculated as follows

$$\phi V_n = 8.15\text{ k/ft }(90\text{ ft}) = 733.5\text{k }(3260\text{ kN}) > 721.5\text{ kip }(3200\text{ kN})$$

$\therefore$ Still, no collector is required

With or without post-tensioning or the direct compression reduction, the concrete diaphragm along Grids 1 and 2 does not require additional collector reinforcement when analyzed as a standard concrete element. This is a significantly different answer than what was generated using a wood diaphragm design methodology. It is obviously the engineer's decision on what method conforms to their comfort level, but it has always baffled the author on why every other concrete element is designed using accepted methods, expect for the largest concrete beam on the project.

Concrete is the strongest diaphragm in building construction. Plywood and metal decking supported by wood and steel beams are pieces and parts, while a cast-in-place concrete floor is effectively a solid membrane. The strength and integrity of the concrete diaphragm is only increased by the post-tensioning. Tendons in a typical structure are the closest thing to continuous reinforcement engineers have at their disposal. While nonprestressed reinforcement is required to be lapped and

Fig. 11—Post-tensioned flat plate diaphragm.

steel/wood beams have connections at columns and girders, a post-tensioning tendon can effectively go slab edge to slab edge as a single piece of reinforcement. The tendons in Fig. 11 show the continuity of post-tensioning in a typical flat plate structure as it runs parallel to a perimeter shear wall. The pre-compression force from the tendon is present from day one and does not require additional load or deflections to generate enough strain to activate the reinforcement. Continuous, active post-tensioning reinforcement is ideal for chords and collectors.

The minimal to no additional collector reinforcement has been the author's experience in the design of millions of square feet of post-tensioned concrete buildings. This was also the experience of my predecessors that educated young engineers like the author on how to design concrete diaphragms. While there is nothing wrong with the wood methodology, and it absolutely makes sense for wood buildings, it is the author's opinion that it should not become the prescriptive method for concrete diaphragm design in future editions of ACI 318.

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