pti Technical Notes

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Effective Width and Post-Tensioning

By Bijan O. Aalami[1]

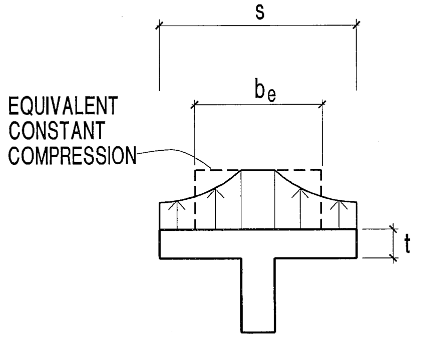
1 - Introduction and Scope

In modeling complex structures, the *effective width* concept is frequently used to arrive at simple representations, amenable to solution through customary formulas. The following describes the widely used effective width notions in modeling concrete structures, in particular post-tensioned members. It concludes that, depending on the structure and actions at hand, different effective width values and treatments may apply.

The *effective width* concept is generally used in modeling , when it becomes necessary to assign a portion of the member's cross-section to represent that member's entire response to the applied loading. The substitute cross-section is selected, such as to yield an end-effect analogous to that obtained from a rigorous analysis of the prototype. The *effective width* approach reduces the computation of a more complex structure into that of a simple beam or frame. The definition and magnitude of the effective width depends on the parameter being analyzed.

2 - Bending of Ribbed Slabs and T-Beams

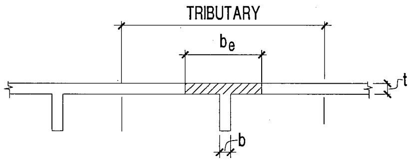
A ribbed slab under uniaxial bending is often treated as a T-beam. From elastic theory, a T-beam subjected to a variable bending moment has a nonuniform distribution of compression in its flange (Fig. 1). Maximum stress occurs at the connection to the web. The magnitude of stress falls with distance from the web. The reduction in stress with distance from the web is due to the *shear lag* (Timoshenko 1970, Girkman 1963) phenomenon. The effective width, b_e (Fig. 1), is defined by most users as the width of an equivalent stress block which, with the maximum value of stress, generates the same total force as the elastic solution. For a beam and slab construction, the distribution of the service load compression and the associated effective width are illustrated in Fig. 2. The area of the hatched block is equal to the elastic distribution over the entire stem's tributary (shown dotted).



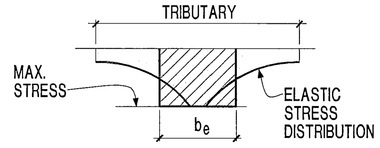
NONLINEAR STRESS DISTRIBUTION IN T-BEAM

Figure 1

The stress reduction is a function of the relative dimensions of the beam's stem and flange, as well as the distribution of moment along the beam. As a result, the effective width for bending is not a constant value. Fig. 3 is an illustration of the effective width for a simply supported beam of infinitely wide flange under different loading conditions. Observe that for a sinusoidal distribution of loading the effective width is uniform.



(a) BEAM AND SLAB CROSS-SECTION



(b) DISTRIBUTION OF STRESS IN FLANGE

BEAM AND SLAB CONSTRUCTION WITH VARYING MOMENT ALONG BEAM

Figure 2

The (16t+b) expression of ACI-318 (ACI, 1992) for effective width of nonprestressed sections is to account primarily for the shear lag phenomenon. For prestressed sections, some engineers use a higher value, typically (24t+b). The selection of a higher effective width is based on the argument that the cracking consideration implicit in ACI's (16t+b) stipulation is less significant in service loading of prestressed members. This rationale is true for service conditions. At limit state howev-

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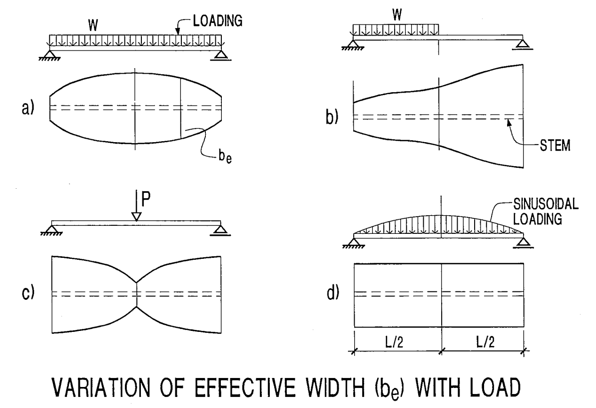


Figure 3

er, the prestressed and nonprestressed sections have a similar effective width response possibly closer to (24t+b). ACI's (16t+b) stipulation was put forth prior to inclusion of limit state in the code.

The implication of effective width concept for service load stresses is important in prestressed members, particularly in ribbed-plate post-tensioned slabs-on-ground (SOG). Fig. 4 shows a typical ribbed slab-on-ground with a ratio of slab thickness to rib spacing of about 40. This ratio by far exceeds the ACI's value of 16, or some design consultants' choice of 24 for one-way prestressed sections. However, from a structural design standpoint, SOGs are two-way systems. For two-way prestressed systems ACI (ACI 1992) stipulates a lesser allowable tensile stress ($6\sqrt{f'c}$ psi; $0.5\sqrt{f'c}$ MPa) than one-way systems ($12\sqrt{f'c}$ psi; $1.0\sqrt{f'c}$ MPa). Although the effective width phenomenon is pronounced in SOG, it is permissible to disregard it in design, and revert to a smeared uniform distribution, in order to retain consistency with the design procedure of elevated post-tensioned two-way slab systems, where the lesser permissible tensile stress is used.

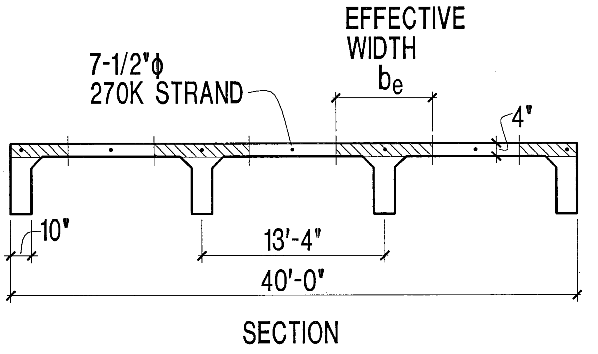


Figure 4

3 - Bending of Two-Way Slab Systems Under Gravity Loading

Two-way slab systems supported on columns (Fig. 5) are frequently modeled as intersecting plane frames (Aalami 1990b, 1989a, Vanderbilt et al 1983). A *simple* plane frame modeled strictly on the cross-sectional geometry of the members' tributary lacks the load carrying contribution of the *twisting moments* inherent in the two-dimensional prototype. Such a *simple* plane frame draws a larger bending moment to the columns than the two-dimensional prototype. In order to obtain a more realistic distribution of bending moment between columns and slab, a frame model must account for the contribution of the twisting moments.

(i) One option is to widen the model frame to an *effective width* beyond its tributary - enough to achieve the correct slab moment at the slab-column junction.

(ii) Another option is to assume the slab's effective width to be equal to its tributary, but reduce the stiffness of the column. The *equivalent frame model*ing (EFM) of ACI uses the second option to render a corrected moment distribution. For the same column geometry as the prototype, the EFM, in effect, uses an effective width larger than the tributary (Fig. 5-c)

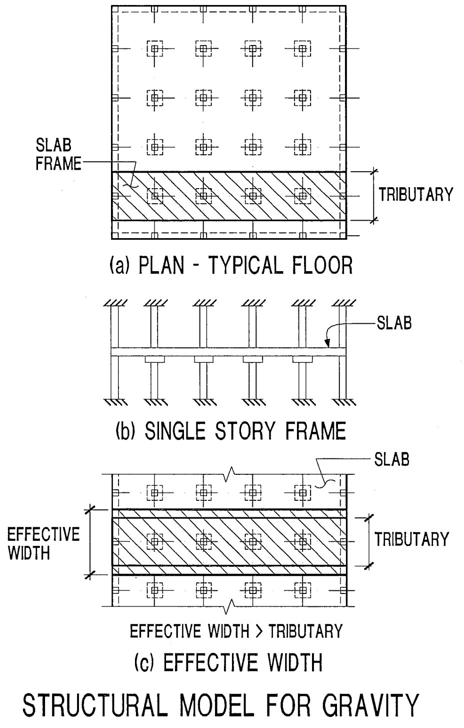


Figure 5

4 - Bending of Slab Frames Under Wind/Seismic Loading

Under lateral loading (Fig. 6), the induced joint moments are shared between the columns and the adjoining slab strip. The rotational stiffness of a concrete slab at a column/slab junction determines the share of moment experienced by the slab. The rotational stiffness of a slab at column-slab junction is less than that given by the gross cross-sectional geometry of the slab's tributary (Mehrain et al 1974, Aalami 1972). Cracking of concrete anticipated under lateral loading results in a further reduction of slab's stiffness. When using a plane frame modeling for the analysis of slabs under earthquake or wind loading, the effective width is generally assumed as 0.33 of the tributary for nonprestressed and 0.5 times the tributary for prestressed slabs.

The existence of two effective width values, one for gravity and one for lateral loading, poses additional effort in computer solution of multistory floor slab constructions. Unlike the linear structures, in this case the stiffness matrices for the two loadings will be different.

5 - Dispersion of Precompression in Prestressed Slabs

The precompression applied to a member is generally introduced at a tendon's anchorage. The precompression disperses into the member and falls in magnitude with distance from the anchorage. For beam

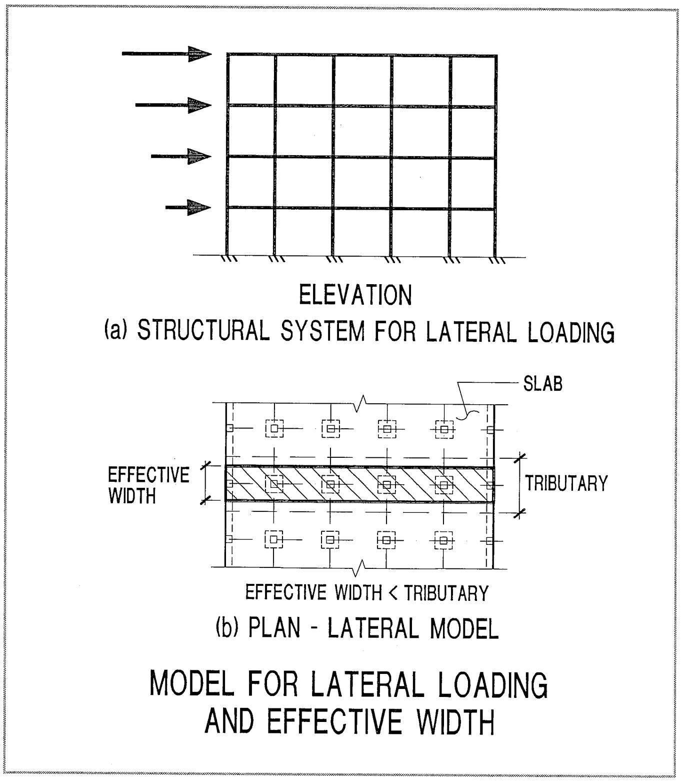


Figure 6

and slab construction (Fig. 7) of dimensions common to parking structures, where post-tensioning is introduced at the beam stem, and where beams are closely spaced relative to their lengths, elastic membrane stress solutions indicate a steep spread of precompression into the slab. For common parking structure geometry having repetitive beams of essentially similar construction and prestressing, the 45 degree dispersion assumed in many simplified analyses is a conservative approximation. A 60 degree distribution is more realistic. From the edge of the slab, at distances greater than the beams' spacing the precompression becomes practically uniform across the entire cross-section of the beam-slab combination.

The wedge shaped regions of slab between the beam stems (Fig. 7-b) do not receive a precompression as experienced by the regions farther away from the slab edge. These wedge regions require special consideration when average precompression is a design parameter.

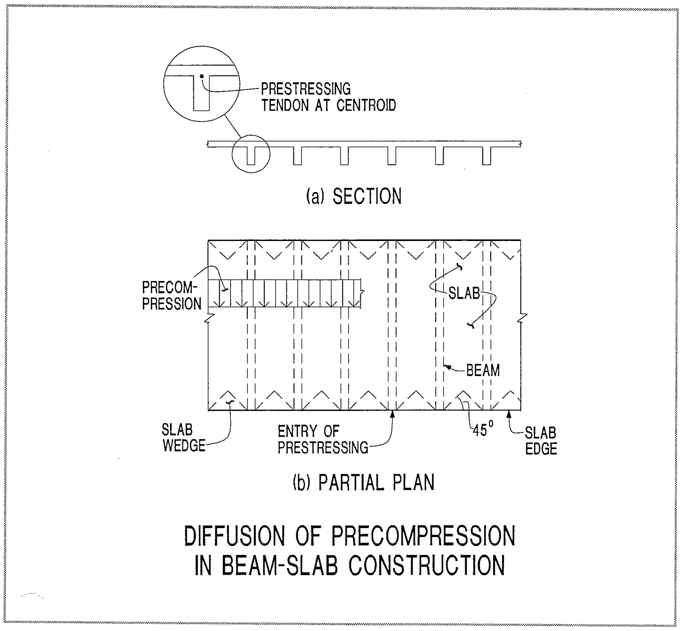


Figure 7

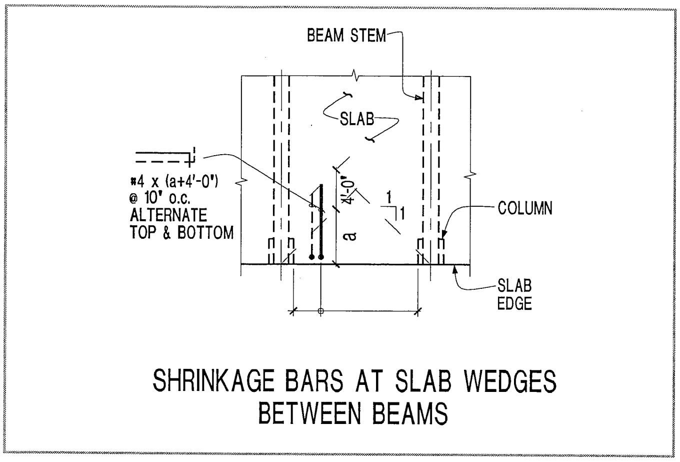


Figure 8

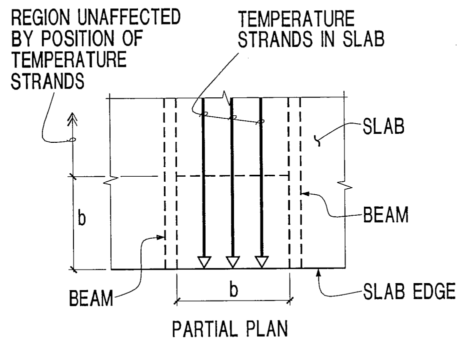
The amount of post-tensioning in a beam is generally governed by its flexural capacity under gravity loading, for which an effective width is assumed for flexure (described earlier) Since the post-tensioning diffuses to a uniform distribution away from the beams' ends, the precompression available in stem reduces with distance from the beam end. On the other hand, the central regions of slabs between the beam stems and outside the effective width areas receive a significant precompression diverted to them from the beam stems. The extreme fiber stress at mid-span is given by:

Stress = Percompression/Area + (Total Moment)/(Section Modulus)

The noteworthy point is that the, *Area*, refers to the *tributary area*, whereas the *Section Modulus* refers to that obtained from the section reduced by *effective width in bending*.

For shrinkage reinforcement of one-way slab systems, such as shown in Fig. 7, ACI prescribes either 100 psi (0.70 MPa) average precompression, or nonprestressed reinforcement equal to 0.0018 times the slab cross-section. For the condition illustrated in Fig. 7, it is only the wedges extending from the slab edge a distance equal to beam spacing which do not experience the stipulated precompression. For the slab wedges, a shrinkage reinforcement detail such as shown in Fig. 8 is necessary. In some practice, straight slab tendons approximately 6 ft (1.8 m)) apart and parallel to the beam are placed in the slab to control temperature and shrinkage (Fig. 9). Note that, for unbonded straight tendons greased and wrapped in plastic, the location and spacing of tendons have no impact on the local shrinkage and temperature response of slab away from the edge. Equal number of tendons bundled together or placed individually provide the same average precompression at regions away from slab edge. For this reason, the effectiveness on central slab regions, of temperature and shrinkage straight tendons placed in slab and between the beams is questionable. Such tendons would be utilized more effectively, if placed and profiled in the beam stems.

It is noteworthy that the code specified nonprestressed shrinkage reinforcement for grade 60 steel (0.0018 of cross-sectional area) develops, at yield, an average force nearly equal to the alternative 100 psi (0.70 MPa) precompression. Where the combination of the two provisions are used, a linear interaction relationship must be employed for shrinkage and temperature adequacy.



BEAM-SLAB CONSTRUCTION WITH SHRINKAGE/TEMPERATURE STRANDS

Figure 9

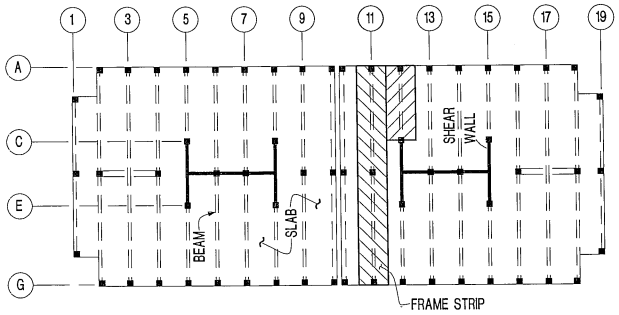
Other measures regarding crack formation and mitigation are discussed in a Post-Tensioning Institute Publication (Aalami et al 1989).

6 - Shortening calculations of Prestressed Members

In shortening computation of post-tensioned beams and slabs, creep is one of the primary parameters. Creep is a direct function of the precompression in the member. At an anchorage zone and its immediate vicinity a member is subject to a higher precompression. But for the bulk of a member, the average precompression is at the level of force divided by the entire tributary. For members reinforced with unbonded tendons, where the creep is related to the average precompression, the *effective width* applicable for shortening calculations is that of the member's entire tributary. The T-beam effective width defined for bending, or the wider-than-tributary effective width implicit in the two-way slab systems do not apply to the shortening computations for creep.

Apart from effective width consideration, isolated modeling of floor regions for shortening calculations poses other questions. Where due to irregular support layout, openings and other features, adjacent frames are highly nonsymmetrical, such as lines 11and 12 in Fig. 10, the shortening of one frame is greatly influenced by the inplane stiffness of adjacent frames. Also, in the vertical frame, particularly at lower levels the inter-story restraints have noticeable impact on shortening of one another. For these reasons, unlike bridge construction, in building construction the shortening computations are oftentimes inaccurate when using simplified models. For this reason, empirical relationships and crack mitigation schemes (Aalami 1989b) are more common.

The stress reduction is a function of the relative dimensions of the beam's stem and flange, as well as the distribution of moment along the beam. As a result, the effective width for bending is not a constant value. Figure 3 is an illustration of the effective width for a simply supported beam of infinitely wide flange under different loading conditions. Observe that for a sinusoidal distribution of loading that the effective width can be closely approximated as a constant.



FLOOR PLAN - CIRCUS CIRCUS PARKING STRUCTURE RENO

Figure 10

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