pti Technical Notes

1717 West Northern Avenue • Suite 114 • Phoenix, Arizona 85021

Dr. Bijan O. Aalami, Editor *For Professionals Engaged in Post-Tensioning Design* *Issue 10 • September 2000*

PRESTRESS LOSSES IN POST-TENSIONED STRUCTURES

by Gail S. Kelley[1]

1.0 INTRODUCTION

Structural drawings for post-tensioned buildings typically give the post-tensioning requirements in terms of final effective forces, i.e. the jacking force minus all prestress losses. Requirements for calculation of prestress losses vary depending on the type of structure, the project specifications, and local practice. On many structures, it is acceptable to assume a reasonable value for the final effective forces rather than calculate prestress losses. For unbonded tendons with 0.5 in. (12.5 mm) diameter strand, 26.8 to 27 kips (119 to 120 kN) is typically used. Twenty-five kips (110 kN) is typically used for bonded 0.5 in. (12.5 mm) strand; the final effective force is lower because the loss due to friction is higher.

This paper discusses the sources of prestress losses with particular reference to equations developed by Zia et al.(1) for long-term loss calculations. Although the focus of the paper is on post-tensioned applications, much of what is discussed also applies to pre-tensioned members.

2.0 SOURCES OF PRESTRESS LOSSES

The sources of prestress loss for post-tensioning are generally considered to be:

- Friction
- Seating (anchorage) loss
- Elastic shortening of the concrete
- Creep of the concrete
- Shrinkage of the concrete
- Relaxation of the steel

Friction and seating are considered immediate losses since they occur while the tendon is being stressed. Strictly speaking, elastic shortening is an immediate loss, however it is often included with the long-term losses (creep, shrinkage, relaxation) since the calculations are similar. Other factors such as temperature change and deformation of the structure under loading will affect the stresses in a tendon. However, these do not necessarily result in a permanent lowering of the stress in the tendon so are not considered prestress losses. In most cases, deformation of the structure increases the stress in the tendon. The stress change will be the same as that of any nonprestressed steel located at the tendon position.

Precise determination of prestress loss is impractical in most cases because the information available is usually not accurate enough. Prestress losses are dependent on a variety of factors including material properties, the method of curing the concrete, the time of loading, environmental conditions and construction details. In addition, the rate of loss due to one factor, such as relaxation of the tendons, is continually altered by changes in stress due to other factors, such as the shrinkage and creep of the concrete. The rate of creep, in turn, is affected by the change in tendon stress.

In order to allow for these interactions, it would be necessary to do a time-step analysis and calculate losses over given time intervals. The values calculated for the end of each interval would be used as the starting values for the next interval. Typically, however, it is assumed that, except for the loss due to relaxation, the various stress losses are independent from one another. Hence, the loss due to each factor can be calculated separately. The total stress loss in a tendon is assumed to be the sum of the individually calculated losses.

3.0 CALCULATION OF PRESTRESS LOSSES

Long-term losses are usually calculated in accordance with equations developed by Zia et al.(1) In most cases, these equations give a reasonable estimate of losses. They are applicable for prestressed members of normal design with an extreme fiber compressive stress of 350 to 1750 psi (2.4 to 12.0 MPa) under full dead load. The equations were developed through comparison with previously published methods and were correlated with theoretical and experimental data, particularly a study by Hernandez and

---

[1] Senior Project Manager, Facility Engineering Associates

Gamble[2]. They apply to concrete with a unit weight of at least 115 pcf (1842 kg/m^3) and a cylinder strength (f'_c) of at least 4000 psi (27.6 MPa) but not more than approximately 6000 psi (41.4 MPa). Losses due to friction and seating are usually calculated in accordance with equations given in ACI 318.[3]

If prestress losses are significantly greater or smaller than estimated values, service load behavior (deflections, camber, cracking load) and connections may be affected. More accurate calculations are required for simply supported slender members that are sensitive to small changes in deflections. The ACI 318 commentary notes that overestimation of prestress losses may be almost as detrimental to service load performance as underestimation since overestimation of losses can result in greater than expected camber and member shortening.

If losses are greater or smaller than the estimated value in members with bonded tendons, there will be little effect on design strength unless the effective stress after losses, f_{se}, is less than 0.5 times the ultimate strength of the steel, f_{pu}. In members with unbonded tendons, the stress in the tendon at nominal strength, f_{ps}, is a function of f_{se}. The nominal strength of the member will thus vary slightly, depending on how the prestress losses are calculated.

3.1 FRICTION LOSSES

Figure 3.1-1 shows a typical post-tensioning tendon profile. When the jacking force P_j is applied at the stressing end, the tendon will elongate in accordance with the well-known formula:

$$\Delta = \int P_x \, dx/AE_s$$

where: A is the cross-sectional area of the tendon

E_s is the modulus of elasticity of the prestressing steel (typically either 28000 or 28500 ksi [193,050 or 196,500 MPa]).

P_x is the force in the tendon at distance x from the stressing end.

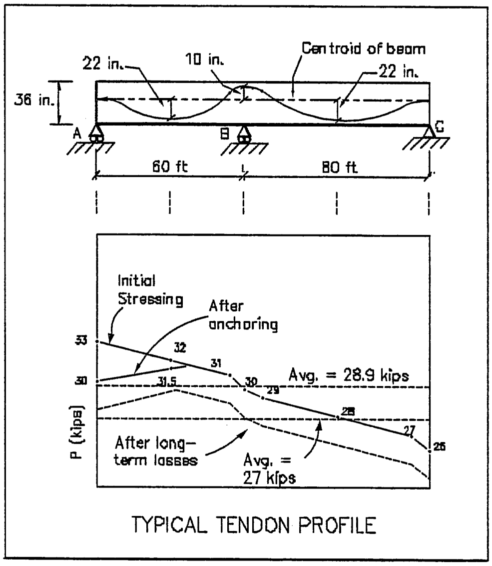


FIGURE 3.1-1

The elongation will be resisted by friction between the strand and its sheathing or duct. As a result of this friction, P_x will decrease with distance from the jacking end. The friction is comprised of two effects: curvature friction which is a function of the tendon's profile, and wobble friction which is the result of minor horizontal or vertical deviations from the intended profile. The loss due to curvature friction will vary along the length of tendon; it is greatest where there is a significant angle change such as at the high points. Table 3.1-1 is a copy of the table of friction coefficients found in the PTI Manual.[4] The table has a range of values for both the μ and K coefficients as well as recommended values which can be used in the absence of actual test results. Note that in this table 'Greased and wrapped' refers to unbonded, monostrand tendons. The ACI 318 commentary includes a similar table. The post-tensioning supplier should be consulted for friction coefficients of duct materials or tendon types not shown in these tables.

RANGE OF VALUES		
Type of Tendon	Wobble coefficient K (per ft)	Curvature coefficient μ (per radian)
Flexible tubing;		
Non-galvanized	0.0005-0.0010	0.18-0.26
Galvanized	0.0003-0.0007	0.14-0.22
Rigid thin wall tubing;		
Non-galvanized	0.0001-0.0005	0.20-0.30
Galvanized	0.0000-0.0004	0.16-0.24
Greased and wrapped	0.0005-0.0015	0.05-0.15
RECOMMENDED VALUES		
Type of Tendon	Wobble coefficient K (per ft)	Curvature coefficient μ (per radian)
Flexible tubing;		
Non-galvanized	0.00075	0.22
Galvanized	0.0005	0.18
Rigid thin wall tubing;		
Non-galvanized	0.0003	0.25
Galvanized	0.0002	0.20
Greased and wrapped	0.0010	0.07

TABLE 3.1-1 FRICTION COEFFICIENTS FOR POST-TENSIONING TENDONS

The equation for calculating the loss of prestress due to friction is:

$$P_x = P_j e^{-(\mu\alpha + Kx)}$$

where:

P_x is the stress in the tendon at point x

P_j is the jacking stress

μ is the coefficient of curvature friction

α is the angle change along the tendon length

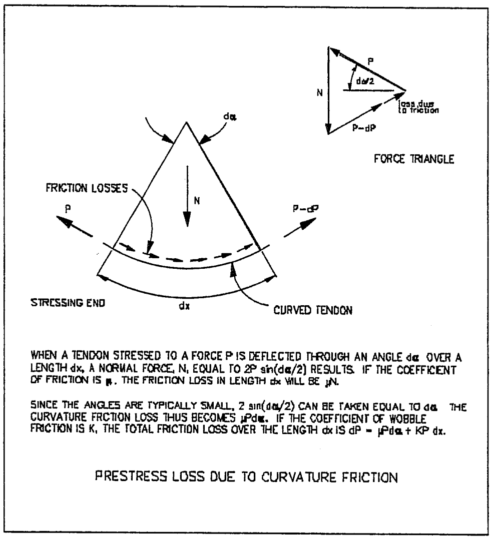
K is the coefficient of wobble friction

x is the length along the tendon.

If ($\mu\alpha + Kx$) is less than 0.3, the equation can be simplified to:

$$P_x = P_j/(1+\mu\alpha + Kx)$$

Figure 3.1-2 is a schematic illustration of curvature friction; Figure 3.1-3 is a schematic illustration of wobble friction.

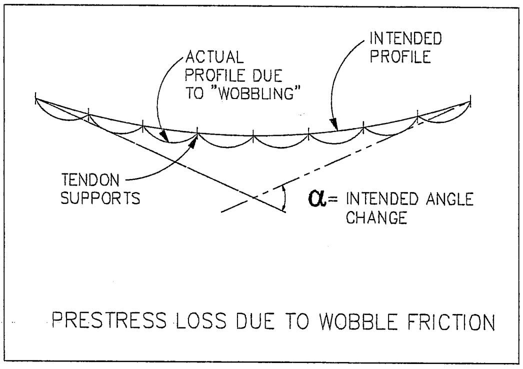


WHEN A TENDON STRESSED TO A FORCE P IS DEFLECTED THROUGH AN ANGLE $d\alpha$ OVER A LENGTH dx, A NORMAL FORCE, N, EQUAL TO $2P \sin(d\alpha/2)$ RESULTS. IF THE COEFFICENT OF FRICTION IS μ. THE FRICTION LOSS IN LENGTH dx WILL BE μN.

SINCE THE ANGLES ARE TYPICALLY SMALL, $2 \sin(d\alpha/2)$ CAN BE TAKEN EQUAL TO $d\alpha$. THE CURVATURE FRCTION LOSS THUS BECOMES $\mu P d\alpha$. IF THE COEFFICIENT OF WOBBLE FRICTION IS K, THE TOTAL FRICTION LOSS OVER THE LENGTH dx IS $dP = \mu P d\alpha + KP\,dx$.

PRESTRESS LOSS DUE TO CURVATURE FRICTION

FIGURE 3.1-2



PRESTRESS LOSS DUE TO WOBBLE FRICTION

FIGURE 3.1-3

3.2 SEATING LOSSES (ANCHOR SET)

Both bonded and unbonded tendons are typically anchored with two-piece, conical wedges. When the tension applied by the jack is released, the strand retracts pulling the wedges into the anchorage device and locks the strand in place. The loss in elongation is small. It depends on the wedges, the jack and the jacking procedure but is typically between 1/8 in. (3 mm) and 3/8 in. (9 mm). This loss in elongation is resisted by friction just as the initial elongation is resisted by friction. To solve directly for the prestress loss, it is necessary to assume a constant friction loss per foot. The prestress loss that corresponds to the measured elongation loss can then be calculated.

More exact calculation is typically done as an iterative process. An anchor set length, L_{set}, is chosen and the loss in force over this length is calculated based on the friction profile. The elongation loss is then calculated by integrating the elongation equation $\Delta = \int P_x \, dx / AE_s$ over the assumed length with the values of P_x calculated from the tendon profile. The anchor set length is adjusted until the calculated Δ is reasonably close to the measured loss of elongation. The stress loss is typically shown on force profile diagrams as the difference between the jacking force and the lock-off force at the stressing end(s) of the member.

3.3 ELASTIC SHORTENING

Elastic shortening refers to the shortening of the concrete member as the post-tensioning force is applied. If there is only one tendon in a member there will be no prestress loss due to elastic shortening since the elastic shortening will have occurred before the tendon is anchored. Typically, however there will be several tendons in a member. As each tendon is tensioned, there will be a loss of prestress in the previously tensioned tendons due to the elastic shortening of the member.

Since an unbonded tendon can slide within its sheathing, it does not experience the same stress-induced strain changes as the concrete that surrounds it. As a result, the average compressive stress in the concrete, f_{cpa}, is typically used for evaluating prestress losses due to both elastic shortening and creep in unbonded tendons. This bases the prestress losses on the average strain along the member, rather than the strain at a particular point.

The equation given for calculating elastic shortening for unbonded tendons is:

$$ES = K_{es} (E_s / E_{ci}) f_{cpa}$$

where:

E_{ci} is the elastic modulus of the concrete at time of prestress transfer

K_{es} = 0.5 for post-tensioned members when tendons are tensioned in sequential order to the same tension. (With other post-tensioning procedures, K_{es} may vary from 0 to 0.5)

f_{cpa} = average compressive stress in the concrete along the member at the center of gravity (cgs) of the tendons immediately after prestressing

Reference 1 indicates that the calculation of elastic shortening for pre-tensioned (bonded) tendons should be based on f_{cpi}, the stress in the concrete at the cgs of the tendons at the critical location along the span (i.e. the point of maximum moment.) It further indicates that there should be consideration of f_g, the stress in the concrete at the cgs of the tendons due to the weight of the structure at the time the prestress is applied. In the typical post-tensioned case however, the tendons are not bonded at the time the prestress is applied. Furthermore, the structure would probably not be self-supporting, thus there would be no moment due to either self-weight or applied load. In such cases, the equation given for unbonded tendons would apply. An exception might be the case of a precast beam that is subsequently post-tensioned. If the beam is self-supporting at the time of prestressing, the effect of stresses due to the self-weight moment plus any other loads the beam is supporting should be considered.

3.4 CREEP

Over time, the compressive stress induced by post-tensioning causes a shortening of the concrete member. The increase in strain due to a sustained stress is referred to as creep. Loss of prestress due to creep is nominally proportional to the net permanent compressive stress in the concrete. The net permanent compressive stress is the initial compressive stress in the concrete due to the prestressing minus the tensile stresses due to self-weight and superimposed dead load moments.

For members with unbonded tendons, the equation is:

$$CR = K_{cr}\,(E_s / E_c)\, f_{cpa}$$

For members with bonded tendons, the equation is:

$$CR = K_{cr}\,(E_s / E_c)\,(f_{cir} - f_{cds})$$

where:

K_{cr} = 1.6 for post-tensioned members

f_{cds} = stress in the concrete at the cgs of the tendons due to all superimposed dead loads that are applied to the member after prestressing.

$f_{cir} = K_{cir} * f_{cpi} - f_g$

where:

f_{cir} = net stress in the concrete at the cgs of the tendons immediately after prestressing and removal of shoring

K_{cir} = 1.0 for post-tensioned members

f_{cpi} = stress in concrete at cgs of tendons due to prestressing forces immediately after prestress has been applied

f_g = stress in concrete at center of gravity of tendons due to weight of structure

As noted above, unbonded tendons do not experience the same strains as the surrounding concrete. An unbonded tendon is typically a single, greased strand encased in a plastic sheathing and anchored at either end of the member. Since there is no connection between the concrete and steel along the length of the tendon, it is reasonable to base the prestress loss due to creep on the average stress in the concrete. Note that the average compressive stress immediately after stressing, f_{cpa}, is used for unbonded tendons, there is no adjustment for the tensile stresses due to self-weight or superimposed dead load.

A bonded tendon is typically comprised of two or more bare strands enclosed in either a plastic or galvanized steel duct. After the strands are stressed, the duct is filled with a cementitious grout. This bonds the strand to the duct; the duct is bonded to the surrounding concrete via both adhesion and deformations on its exterior surface. Once the duct is grouted, the shortening of the concrete member due to creep will result in a comparable shortening (loss of elongation) in the tendon. This loss of elongation will result in a loss of prestress that varies along the length of the tendon according to the local stress in the concrete.

3.5 SHRINKAGE

When calculating prestress losses, shrinkage is considered to be solely due to water loss. The shrinkage strain is therefore a function of the member's volume/surface ratio and the ambient relative humidity. The prestress loss due to shrinkage is obtained by multiplying the effective shrinkage strain by a factor which accounts for the shrinkage which will have taken place before the prestressing is applied. The effective shrinkage strain, e_{sh}, is obtained by multiplying the basic ultimate shrinkage strain, $(e_{sh})_u$, taken as 550×10^{-6}, by the factors (1 - 0.06 V/S) and (1.5 - 0.015RH). Thus

$$\begin{aligned} e_{sh} &= 550 \times 10^{-6}\,(1 - 0.06\,V/S)(1.5 - 0.015RH) \\ &= 8.2 \times 10^{-6}\,(1 - 0.06V/S)\,(100 - RH) \end{aligned}$$

where: V/S is the volume to surface ratio and RH is the relative humidity expressed as a percent.

The equation for prestress loss due to shrinkage is:

$$SH = 8.2 \times 10^{-6}\,K_{sh}\,E_s\,(1 - 0.06\,V/S)\,(100 - RH)$$

The factor K_{sh} accounts for the shrinkage that will have taken place before the prestressing is applied. For post-tensioned members, K_{sh} is taken from the following table:

DAYS	1	3	5	7	10	20	30	60
K_{sh}	0.92	0.85	0.80	0.77	0.73	0.64	0.58	0.45

TABLE 3.5-1 SHRINKAGE CONSTANT K_{sh}

'Days' is the number of days between the end of moist-curing and the application of the prestress. In structures that are not moist-cured, K_{sh} is typically based on when the concrete was cast. In most structures, the prestressing is applied within five days of when the concrete is cast, whether or not it is moist-cured. Note that the effective shrinkage strain is zero under conditions of 100% relative humidity, i.e. if the concrete is continuously submerged in water.

3.6 RELAXATION OF TENDONS

Relaxation is defined as a gradual decrease of stress in a material under constant strain. In the case of steel, relaxation is the result of a permanent alteration of the grain structure. The rate of relaxation at any point in time depends on the stress level in the tendon at that time. Because of other prestress losses, there is a continual reduction of the tendon stress; this causes a reduction in the relaxation rate.

The equation for prestress loss due to relaxation of the tendons is:

$$RE = [K_{re} - J(SH + CR + ES)]C$$

The factor J accounts for the reduction in tendon stress due to other losses. The basic relaxation value, K_{re}, and J are a function of the type of steel. C is a function of both the type of steel and the initial stress level in the tendon (f_{pi}/f_{pu}). Table 3.6-1 shows values of K_{re} and J for different types of steel. Table 3.6-2 gives values for C based on (f_{pi}/f_{pu}). The factor C accounts for the fact the ratio of yield strength to ultimate strength, f_{py}/f_{pu}, is different for stress-relieved (normal relatation) and low-relaxation strand.

Although ACI 318 allows a stress of 0.74 f_{pu} along the length of the tendon immediately after prestress transfer, the stress at post-tensioning anchorages and couplers is limited to 0.70 f_{pu}. The ratio f_{pi}/f_{pu} is typically taken as 0.70 for post-tensioned members unless more exact calculations are done. With short tendons however, the loss due to anchorage set may be such that a lower f_{pi}/f_{pu} ratio should be used.

	Grade and type*	K_{re}	J
STRESS RELIEVED (NORMAL RELATATION)	270 strand or wire	20000	0.15
	250 strand or wire	18500	0.14
	240 wire	17600	0.13
	235 wire	17600	0.13
	160 bar	6000	0.05
	145 bar	6000	0.05
LOW RELAXATION	270 strand	5000	0.040
	250 wire	4630	0.037
	240 wire	4400	0.035
	235 wire	4400	0.035

* In accordance with ASTM A416, ASTM A421, ASTM A722

TABLE 3.6-1 STRESS RELAXATION CONSTANTS K_{re} AND J

f_{pi}/f_{pu}	Stress Relieved (Normal Relatation) Strand and Wire	Stress Relieved Bar and Low Relaxation Strand and Wire
0.80		1.28
0.79		1.22
0.78		1.16
0.77		1.11
0.76		1.05
0.75	1.45	1.00
0.74	1.36	0.95
0.73	1.27	0.90
0.72	1.18	0.85
0.71	1.09	0.80
0.70	1.00	0.75
0.69	0.94	0.70
0.68	0.89	0.66
0.67	0.83	0.61
0.66	0.78	0.57
0.65	0.73	0.53
0.64	0.68	0.49
0.63	0.63	0.45
0.62	0.58	0.41
0.61	0.53	0.37
0.60	0.49	0.33

TABLE 3.6-2 STRESS RELAXATION CONSTANT C

3.7 APPROXIMATE LOSSES

Tendons are typically stressed at the maximum allowed by ACI 318, 0.8 * f_{pu} = 216 ksi (1490 MPa). In the absence of any calculations, the average stress along the tendon after seating is taken as 0.7 * f_{pu} = 189 ksi (1300 MPa) for unbonded tendons. The final effective stress is then assumed to be 175 ksi (1205 MPa). This is a total prestress loss of 41 ksi (280 MPa) (19%) and results in a final effective force of 26.8 kips (119 kN) for 0.5 in. (12.5 mm) strand. For convenience, this is often rounded to 27 kips (120 kN).

The relative percentages of the various factors contributing to prestress losses will vary. Beams are usually designed with a higher precompression than slabs, however they have a higher volume to surface ratio. The losses due to elastic shortening and creep are therefore higher in beams than slabs; the loss due to shrinkage is higher in slabs than beams. For most tendons, the anchor set has only a minor effect on the final effective forces. On tendons which are less than 20 ft. (6.1m) long however, the loss due to seating will be large enough that the final effective force in the tendon is considerably less than 27 kips (120 kN). This needs to be taken into account when designing members with short tendons.

3.8 EFFECTS OF RESTRAINT

The prestress losses discussed above (friction, seating, elastic shortening, creep, shrinkage and relaxation) cause a reduction of stress in the tendon. This, in turn, results in a reduction of the precompression applied to the concrete. The precompression in a member may also be affected by connections to other structural members such as stiff shear walls that restrain its movement.

Losses due to restraint can be both immediate and long-term. If a prestressed member is rigidly attached to another member, the adjoining member will absorb part of the prestress force during stressing. If the prestressed member cannot move freely to accommodate volume changes due to subsequent temperature changes and shrinkage and creep of the concrete, there will be further transfer of force to the restraining member.

Potential loss of prestress to adjoining members must be evaluated during design. The stresses in the concrete should be calculated based on rational procedures that consider relative stiffnesses, equilibrium of forces and strain compatibility. Losses due to restraint are discussed further in Aalami et al. (5)

3.9 LIGHTWEIGHT CONCRETE

The basic ultimate shrinkage strain for sand-lightweight concretes may be significantly higher than the value given in Section 3.5. As a result, the prestress loss due to shrinkage will be higher. In addition, sand-lightweight concrete has a considerably lower modulus of elasticity than normal-weight concrete. This will result in a higher loss due to elastic shortening. Since the residual stress in the tendon is lower however, the loss due to creep will be lower. The creep coefficient K_{cr} is usually reduced by 20% for sand-lightweight concrete to account for this. The creep behavior of prestressed members made with all lightweight concrete should be based on the properties of the particular aggregate since there are considerable variations.

4.0 CONCLUSIONS

This Technical Note discusses the sources of prestress losses with particular reference to the equations developed by Zia et al. Initially, ACI 318 proposed lump sum values for long-term prestress losses in order to ensure that bidding was done fairly with equivalent final effective forces. Similar lump sum values were adopted by AASHTO. With the publication of 318-83, however, ACI 318 indicated that lump sum values for prestress losses were considered obsolete. It further indicated that reasonably accurate estimates could be obtained using the equations developed by Zia et al. An understanding of the sources of prestress loss will allow the engineer to determine if these estimates are acceptable or if more exact calculations are required.

REFERENCES

1. Zia, P., Preston, H.K., Scott, N.L. and Workman, E.B., "Estimating Prestress Losses," Concrete International, June 1979, pp. 32-38.
2. Hernandez, H.D. and Gamble, W.L., "Time Dependent Prestress Loses in Pretensioned Concrete Construction ," Structural Research Series, No. 417, Civil Engineering Studies, University of Illinois, Urbana, May 1975, 171 pp.
3. ACI Committee 318, "Building Codes Requirements for Reinforced Concrete (ACI 318-99)," American Concrete Institute, Detroit, 1999.
4. PTI Manual, 5th Edition, Post-tensioning Institute, Phoenix, Arizona, 1990
5. Aalami, B., and Barth, F. "Restraint Cracks and Their Mitigation in Unbonded Post-Tensioned Building Structures," Post-Tensioning Institute, 1988, 50 pp.

FOOTNOTE:

The commercially available computer program FELT uses the equations in ACI 318 and those developed by Zia et al. to determine prestress losses. Numerical examples based on these equations are given in the FELT manual. More accurate formulations for prestress losses in tendons and losses in prestressed members have been developed over the last forty years at the University of California Berkeley as part of a long-term coordinated research project. The results of this research are made available for practicing engineers through the software ADAPT-ABI. Numerical examples are provided in the ADAPT-ABI manual.

ADAPT-FELT Manual, "Friction, Elongation and Long-Term Stress Losses in Prestressing," ADAPT Corporation, Redwood City, Ca, 1999, 103 pp.

ADAPT-ABI Manual, "Segmentally Constructed Frames and Bridges," ADAPT Corporation, Redwood City, Ca, 1999, 232 pp.

POST-TENSIONING INSTITUTE
Technical Notes
ISSUE 10 • SEPTEMBER 2000
1717 W. NORTHERN AVENUE, SUITE 114, PHOENIX, ARIZONA
(602) 870-7540 FAX (602) 870-7541
www.post-tensioning.org

Printed on recycled paper. PRINTED IN U.S.A. CCI 2 M © PTI 9/00