pti Technical Notes

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"THICKER IS WEAKER?"

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PROBLEM

The author has occasionally encountered the erroneous opinion, generally expressed by those unfamiliar with prestressed concrete fundamentals, that building a post-tensioned slab-on-ground thicker than its designed thickness (placing more concrete in the slab than required) reduces its resistance to shrinkage and temperature cracking. This capacity reduction, it is suggested, results from the reduction in prestress compressive stress (the prestress force divided by the cross-sectional area of the slab F/A) in the thicker slab. In other words, for post-tensioned slabs, thicker is weaker.

DISCUSSION

This opinion is intuitively and mathematically incorrect. All other things being equal, and as common sense would dictate, thicker post-tensioned slabs are more resistant to applied stress than are thinner slabs. This includes both applied axial stress (as produced by shrinkage, creep, and temperature effects) and bending stress (as produced by transverse loading). It seems hardly necessary to demonstrate, with mathematics and fundamental structural engineering principles, something that is so intuitively obvious, but it can be done with a few relatively simple examples. In these examples, two post-tensioned slab-on-ground models will be used, one 5 in. thick, the other 10 in. thick. Concrete tensile stresses are considered positive, compression stresses negative. Pertinent properties for the two slab models are tabulated below.

First, consider the 5 in. thick post-tensioned slab-on-ground shown in Fig. 1. The freebody diagram studied is a portion of the slab to the left of Section Z-Z. The prestress force F of 5000 lb/ft is roughly equivalent to ½ in. diameter tendons spaced 5 ft on center.

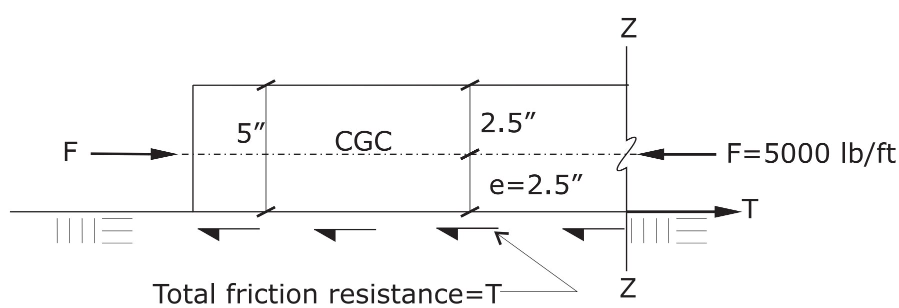

Fig. 1 - 5 in. Thick Post-Tensioned Slab-on-Ground Model

Table 1 - Properties for the Slab Models

Slab Thickness T (in)	Cross-Sectional Area A (in^2)	Section Modulus S (in^3)	Modulus of Rupture f_r (psi)	Compressive Strength f'_c (psi)	Prestress Force F (lb/ft)
5	60	50	375	2500	5000
10	120	200	375	2500	5000

The average axial prestress compressive stress on the slab cross-section is F/A = 5000/60 = 83 psi. To produce a condition with incipient cracking, a tensile force T, parallel to the long axis of the slab, must be applied sufficient to produce a flexural tensile stress on Section Z-Z equal to 7.5 $\sqrt{f'_c}$ = 375 psi, the modulus of rupture, at an extreme fiber. The tensile force is applied to the slab in the form of friction between the soil and the slab, which resists the axial shortening. The magnitude of T is a direct measure of the resistance of the slab to cracking caused by volume changes, such as those produced by shrinkage, creep, and temperature change. T can be calculated as follows:

The tensile stress f_r at the bottom of the slab is:

$$f_r = -\frac{F}{A} + \frac{T}{A} + \frac{Te}{S}$$

$$f_r + \frac{F}{A} = T\left(\frac{1}{A} + \frac{e}{S}\right)$$

Solving for T:

$$T = \frac{f_r + \frac{F}{A}}{\frac{1}{A} + \frac{e}{S}} = \frac{375 + \frac{5000}{60}}{\frac{1}{60} + \frac{2.5}{50}} = 6875 \text{ lb/ft}$$

While peripheral to the point of this article, it is of interest to note that this analysis demonstrates that tensile cracking caused by restraint to shortening in ground-supported slabs generally initiates at the bottom of the slab and propagates upwards.

Now consider a slab with the same number, spacing, and location of post-tensioned tendons, same concrete material properties, but twice as thick as shown in Fig. 2. The additional concrete is logically assumed to be placed above the surface of the original 5 in. thick slab, with the dimensions between the tendon and the ground surface remaining unchanged (the tendon profile would be established by the designed slab thickness, rather than the unanticipated thicker slab).

For the 10 in. thick slab, the average axial prestress compressive force is F/A = 5000/120 = 42 psi. In a similar manner, the tensile force T required to produce cracking can be calculated as:

$$T = \frac{f_r + \frac{F}{A} + \frac{Fe_1}{S}}{\frac{1}{A} + \frac{e_2}{S}}$$

$$T = \frac{375 + \frac{5000}{120} + \frac{5000 \times 2.5}{200}}{\frac{1}{120} + \frac{5}{200}} = 14,375 \text{ lb/ft}$$

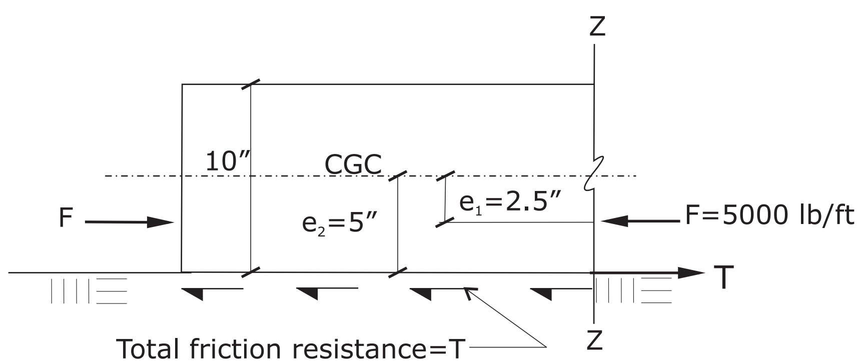

Fig. 2 - 10 in. Thick Post-Tensioned Slab-on-Ground Model

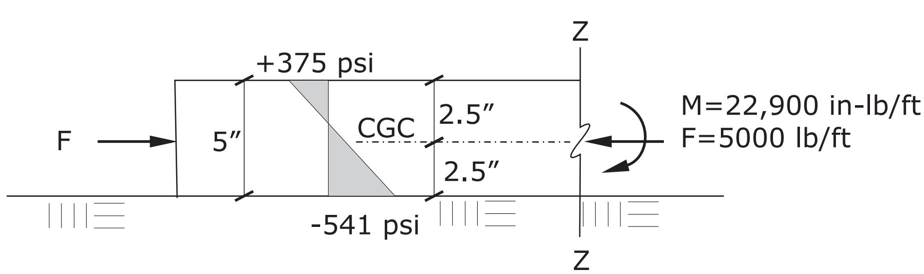

Fig. 3 - Flexural Stresses in the 5 in. Thick Slab-on-Ground Model

Thus in spite of the reduced prestress compression stress, the thicker slab still provides more than twice the resistance to cracking from axial shrinkage, creep, and temperature effects than does the thinner slab (14,375/6875 = 2.1).

To address bending resistance, the effects of axial shortening and subgrade friction will be ignored, focusing only on the bending behavior of the slab. Consider the same 5 in. slab shown in Fig. 1, but now with an applied moment on the slab cross-section of 22,900 in-lb/ft, selected because it precisely produces a maximum flexural tensile stress in the concrete of 375 psi, the modulus of rupture, and thus represents a condition of incipient flexural cracking:

$$f = -\frac{F}{A} \pm \frac{M}{S} = -\frac{5000}{60} \pm \frac{22,900}{50}$$

$$f = -83 \pm 458 = +375, \text{ -541 psi}$$

If the slab thickness is increased to 10 in., with all other slab parameters unchanged, the resultant flexural stresses are as shown below in Fig. 4.

$$f = -\frac{F}{A} \pm \frac{Fe_1}{S} \pm \frac{M}{S}$$

$$f = -\frac{5000}{120} \pm \frac{5000 \times 2.5}{200} \pm \frac{22,900}{200}$$

$$f = -42 \pm 63 \pm 115 = +136, \text{ -220 psi}$$

Thus the bending stresses in the thicker slab, with all other slab parameters equal, are substantially less than those in the thinner slab, even though the average compression stress F/A in the thicker slab is half that of the thinner slab. It could be argued that the increased slab thickness results in increased stiffness and an increase in moment. However, that is only true if the slab curvature remains constant or increases, in accordance with the fundamental equation relating curvature ϕ and moment M:

$$M = EI\phi$$

The increased stiffness will result in a decrease in curvature, and the increase in moment resulting from increased stiffness will tend to be offset by a resultant decrease in curvature. Thus the comparison between 5 in. and 10 in. thick slabs resisting the same moment is valid. Once again, science confirms logic, and we reach the conclusion mathematically that thicker slabs are more resistant to bending stresses than are thinner slabs, all other things being equal. The reduction in axial compression in thicker slabs is more than offset by their increase in section modulus.

An examination of post-cracking behavior (ultimate strength) leads to a similar conclusion. Slabs built thicker than specified by design, all other slab dimensions and properties being equal, will have increased flexural strength in one bending direction (due to the increase in the d dimension, the distance between the centroid of the tendon force and the extreme compression fiber), and an unchanged bending strength in the other. Thus, in the post-cracking condition, there is no downside to building ground-supported slabs thicker than specified.

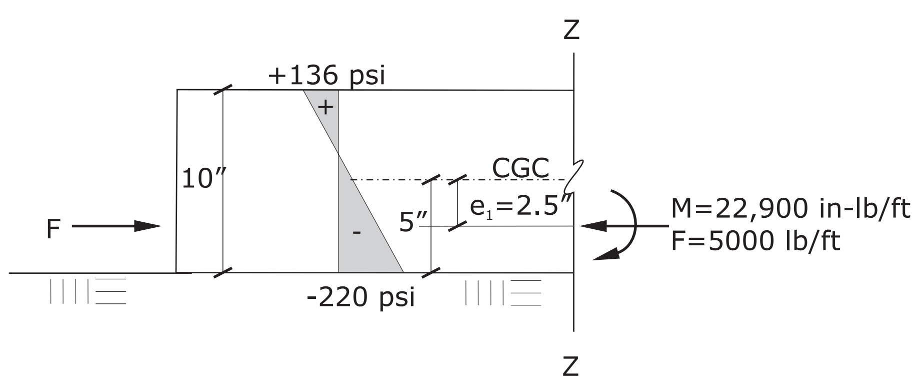

Fig. 4 - Flexural Stresses in the 10 in. Thick Slab-on-Ground Model

The increased weight of thicker slabs is a final consideration. In ground-supported slabs, an increase in weight results in an increase in soil bearing pressure. However this effect is generally negligible since the increased load is applied over the entire slab footprint in which it occurs. In the examples presented the additional 5 in. slab thickness increases the soil pressure by only 63 psf.

CONCLUSION

In conclusion, the above discussion presents numerical examples which demonstrate, under both pre and post-cracked conditions, that ground-supported post-tensioned slabs, built thicker than required by design, are more resistant to applied stress, axial or bending, than if they were built precisely with their design thickness. While this article deals specifically with slabs, the same argument and conclusion can be extended to increases in the width and thickness of ribs in ribbed foundations. In that case, it can be shown that the reduction in compressive stress produced by the increase in rib depth or width is more than offset by the resultant increase in section modulus. The author can conceive no circumstance where this conclusion would not be true.

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