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One-Way and Two-Way Post-Tensioned Floor Systems

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1—INTRODUCTION

This work describes the structural modeling of concrete floor systems, in particular floors reinforced with post-tensioning. It is aimed at the post-tensioning design engineer with three primary objectives in mind, namely, (i) re-state the concepts and discuss the one- and two-way modeling of floor systems, (ii) outline the application of one-way and two-way systems to post-tensioned floors, and, (iii) review the application of the code to structural modeling of post-tensioned floors.

Several simplifications are made in the discussions that follow. These are made to enhance the readability and focus on the subject, without compromising the validity and generality of the concepts. The simplifications are:

Loss of prestressing in tendons due to friction is disregarded. In the bulk of the discussions, it is assumed that the force along a tendon remains constant.

In most of the examples offered, the tendons are assumed unbonded. Unbonded systems are the primary method of construction in the building industry. However, their selection herein is based on ease of material comprehension.

The tendons are assumed to be straight in most of the examples. In reality, post-tensioned tendons are draped (profiled) to provide uplift for serviceability, and to contribute to the strength of the member when overloaded. The profiling does not impact the structural system in the context of this work.

When approaching a failure mechanism, floor slabs develop membrane forces due to finite displacements. Membrane forces, as well as arching action, increase the load carrying capacity of slabs. This work disregards their contribution in the discussion of a slab's strength.

Poisson's ratio relates the moment and curvature of one direction to that of the perpendicular direction. The influence of Poisson's ratio is not included in the main body of the work.

Column size and geometry play a role in the failure mode of a floor supported directly on columns. The effect of column cross-sectional area and size is not covered.

2—DEFINITIONS AND CONCEPTS

2.1 Concept of one-way and two-way system

A simple definition for the one- and two-way structural models of slab systems can be made through reference to the path a load follows, from its point of application to where it reaches the slab supports. Geometry of construction, disposition of reinforcement, prestressing, distribution of loading as well as the magnitude of loading, can affect the load path. An active load path prior to cracking of concrete can change as concrete cracks and the reinforcement becomes mobilized. Further, a slab's natural load path may be different from the path envisioned by the engineer in his/her design. Several load paths may exist to carry the applied loading satisfactorily to the slab supports. It is the structural model associated with a selected load path which determines whether the system for the load path picked is one- or two-way. Hence, it is concluded that one- or two-way categorization is not necessarily an inherent feature of a slab system. Rather, it refers to the path selected for design, or the path used for adequacy verification. If at the strength demand for a structure only one feasible load path exists, the question of path selection becomes mute.

Figure 2.1-1(a) shows a simply supported beam, together with the free body diagram of a cut along the beam's length. The applied loading, F, is carried to the supports, A, and, B, by way of moments and shears indicated on the cut section. The load path is along AB. The vehicle of load transfer is moment and shear. The system is defined as *one-way.*

In part(b) of the same figure, the applied loading, F, is placed on a system of intersecting beams. Observe on the cut of this beam that, to resist the applied loading, F, moments and shears are mobilized in two directions, namely, AB and CD. The system is referred to as two-way. The exact sharing of the load between the two directions, AB, and, CD, depends on a number of factors, such as stiffness of the two beams prior to cracking of concrete, and the amount of rein-

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forcement after cracking. In the framework of the present discussion, the central issue is to select an adequate path, or to verify that for the level of loading given, one or more satisfactory paths exist. In practical design, the exact distribution of the resisting actions (moment, shear, axial force) is not the primary concern, so long that satisfactory resistance of the structure to the applied loading can be ascertained. Obviously, the natural response of the intersecting beams is to carry the load in both, AB, and, CD, directions—that is to say in two ways. However, it is permissible to model the structure as a one-way system, with CD, assigned to carry the entire loading for both in-service and strength limit states. In this case the model is a one-way system. Or, it is permissible that the design engineer assigns fractions of the load which would be carried by each beam. In the latter case the model would be a two-way system.

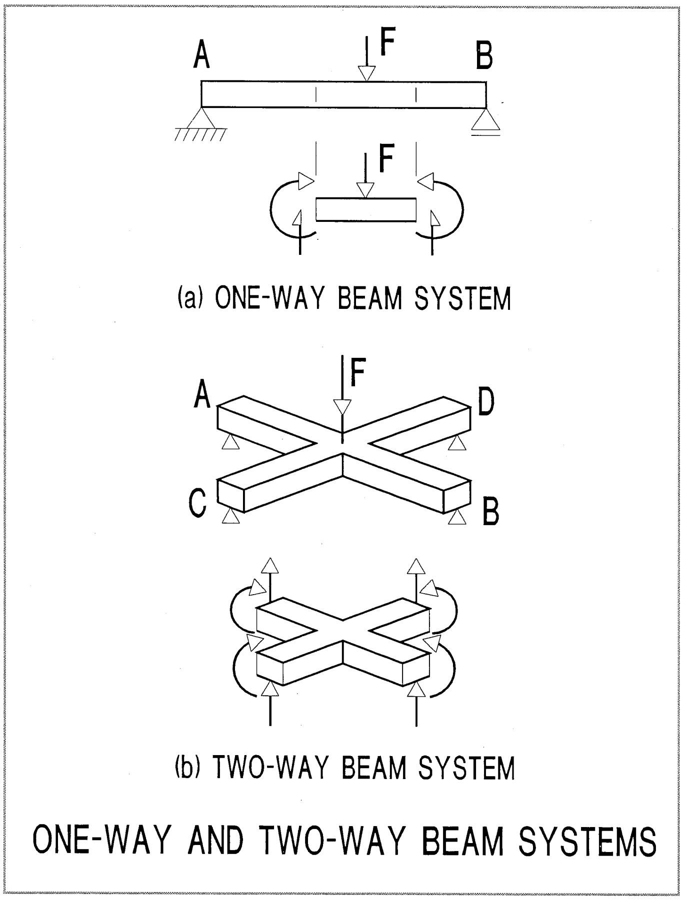


FIGURE 2.1-1

It is true that, in structural modeling, the engineer can assign values for loads to be carried in different directions, and conclude with an acceptable design. However, the closer the engineer's load assignment is to the natural response of a structure, the less cracking the structure is likely to undergo, when called upon to mobilize the load path assigned by the engineer in its design and detailing. This is an important consideration for serviceability behavior of post-tensioned structures, where in-service design is an attempt to control cracking under working conditions. The assignment of loading and selection of load path is not critical for strength limit state, since in most designs the structure is assumed to have cracked extensively and rendered unserviceable when reaching its strength limit.

In summary, in the present treatment of the subject matter, distinction is made between the *natural response* of a structure to loading, its *design load path* and the structure's *analysis load path.*

As a refresher for the discussion that follows, refer to the beam of Figure 2.1-2 and observe that an applied moment, M, causes a curvature, (1/R), where R is the radius of curvature of the bent beam. The moment, M, is proportional to the resulting curvature, (1/R). Generally, for a slab of uniform thickness and properties, the direction of the observed largest curvature is the direction along which the larger moments are transferred. Poisson's ratio plays a minor role in this respect as detailed in Appendix A.

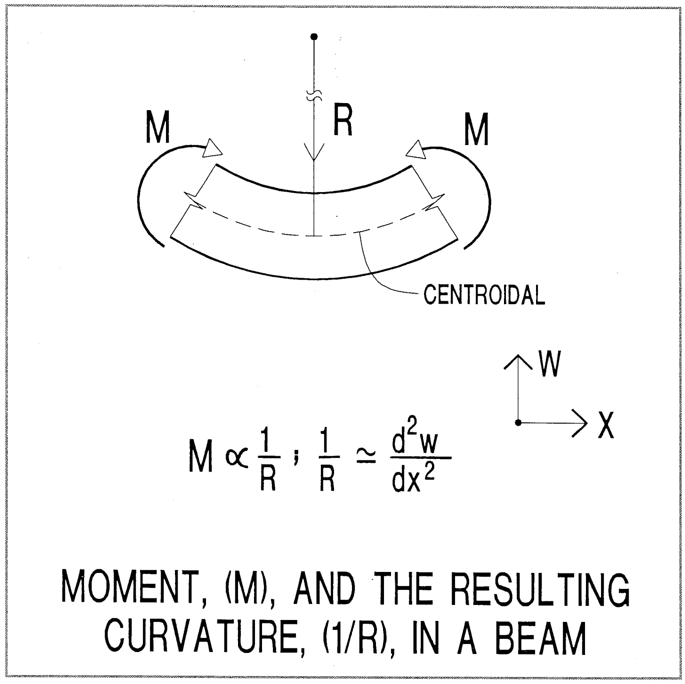


FIGURE 2.1-2

2.2 Load Carrying Characteristic of Slabs

An element of slab acting in two-ways (Fig. 2.2-1(a)) has curvature in both directions. An element of slab acting in one-way develops shear and moments in one-direction (Fig. 2.2-1(b)). Note that the discussions are limited to zero Poisson's ratio, with the understanding that its exclusion does not affect the conclusions arrived at.

Following the preceding argument, when curvature, (1/R), is zero in a given direction, there is no load carrying action in that direction (Fig. 2.2-1(b and d)). For a uniform slab, greater shear and moment are carried in direction of larger curvature.

Consider a square rectangular plate, simply supported on all four sides and loaded with a uniform intensity of loading. For illustration purposes, let the plate be made up of steel and be under its own weight. The plate is homogeneous and has the same stiffness and material properties in both directions. Note in Figure 2.2-2 that the plate flexes in both directions with equal deflections. Hence, it develops a resistance as a two-way action. Due to symmetry, one quarter of the loading is carried to each support side. The stresses at the center of plate, caused by the moments in the two x- and y-directions are shown in Figure 2.2-2(f), with maximum tension stresses at the bottom fiber. The larger the loading, the higher are the stresses. So long as the stresses in the plate remain within the limits of the plate material, the loading on the plate can be increased and the plate retains its two-way load carrying characteristic.

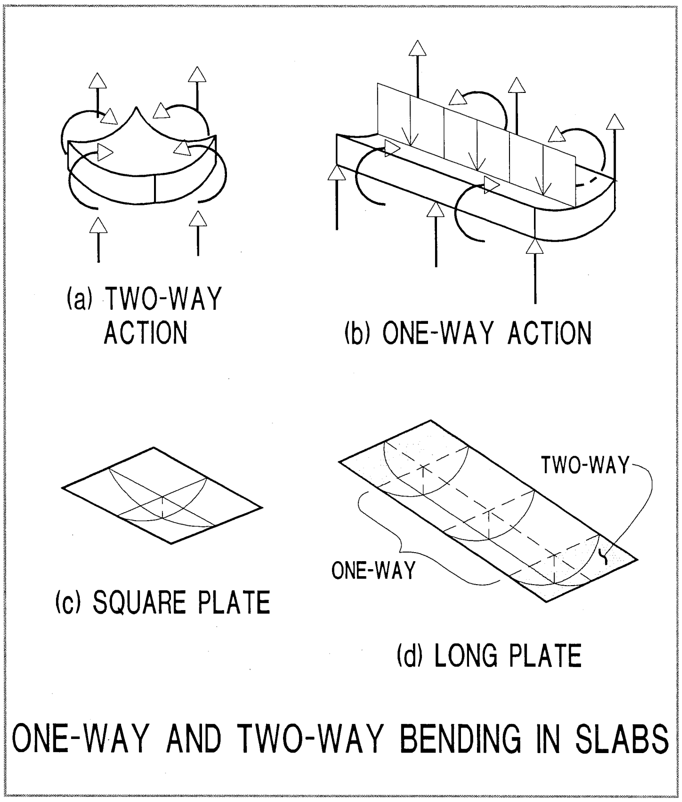


FIGURE 2.2-1

Extend the preceding example to a hypothetical plate made up of plain concrete. If the tensile stresses caused under self-weight (Figure 2.2-2(f)) are less than concrete's tensile stress limit, the plain concrete slab can hold its own weight and would not collapse. Similar to the steel plate, the slab would flex in two-directions and would undergo two-way action. Since the tensile stress limit of concrete is low, this application of concrete is not of practical significance. The plain concrete reaches its load carrying capacity once cracking is initiated, unless reinforcement is added to resist the post-crack loading.

One method of extending the capacity of the plain concrete example is to instill a precompression in the slab. Consider another hypothetical scenario (Figure 2.2-3), in which the plain concrete slab, resting on four wall supports on its perimeter, is subjected to a biaxial precompression. The precompression is induced through a series of jacks placed around the slab's perimeter. The jacks react against a strong ring outside the slab. The tensile stresses developed due to flexing of the slab (Figure 2.2-3(c)) are counteracted by the precompression the jacks impart. The amount of the precompression can be regulated to retain the resulting tensile stresses within the limits of the concrete material used. For the geometry of the example, the slab flexes in both directions and transfers the weight (applied loading) equally to the four wall supports at slab's perimeter (Figure 2.2-3(d)). The response of the slab is a two-way action.

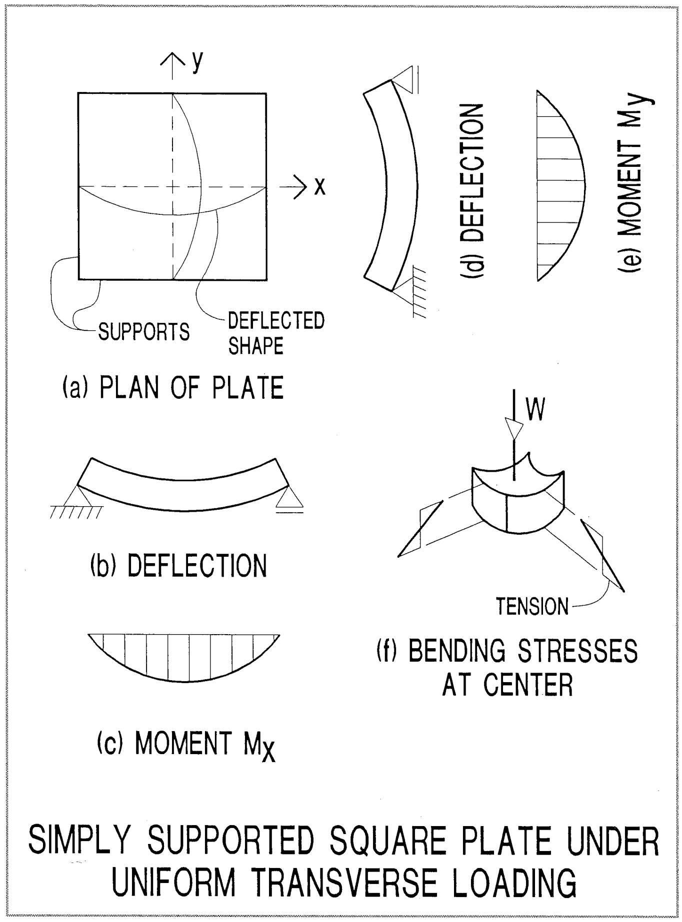


FIGURE 2.2-2

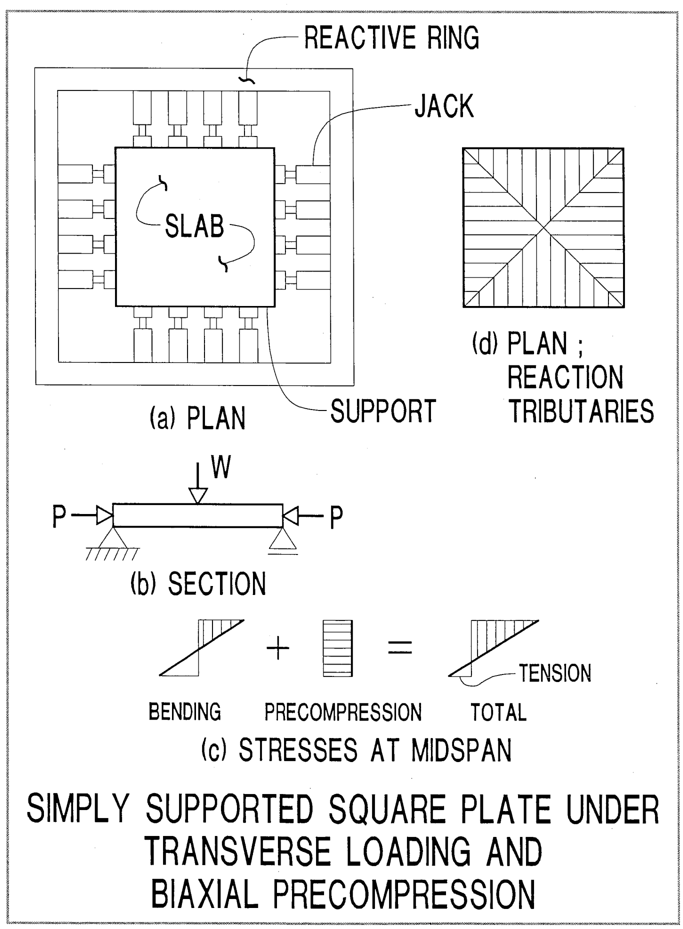
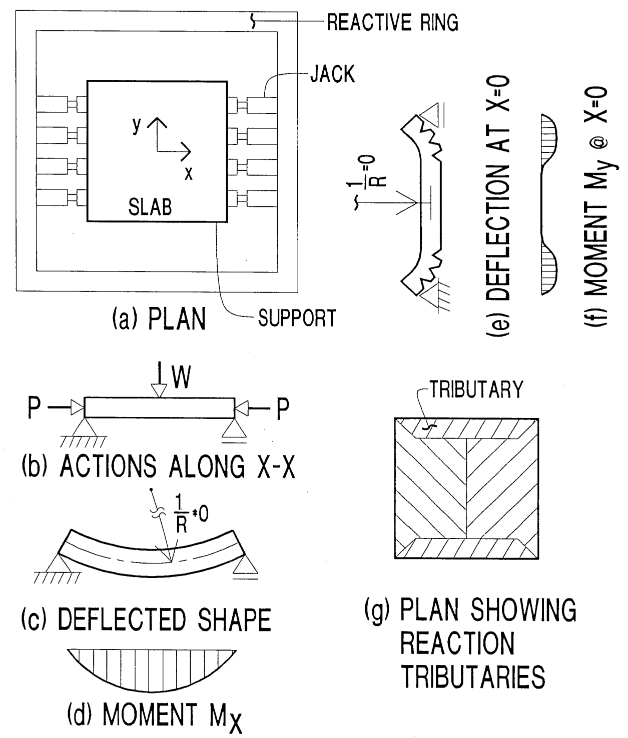


FIGURE 2.2-3

Figure 2.2-4 shows the same slab, but this time subjected to precompression in one-direction (along x-x). Assume loading is increased beyond the cracking limit of concrete in direction where no precompression is provided (y-y). The load would then be carried essentially in direction of precompression (x-x). In y-y direction, the plate deflects as shown in part (e) of the figure. In this direction, the curvature at center of plate is zero. Along the edges parallel to x-direction, the plate cracks to accommodate the deflected shape imposed by bending in x-direction. The response of the slab is a one-way behavior.

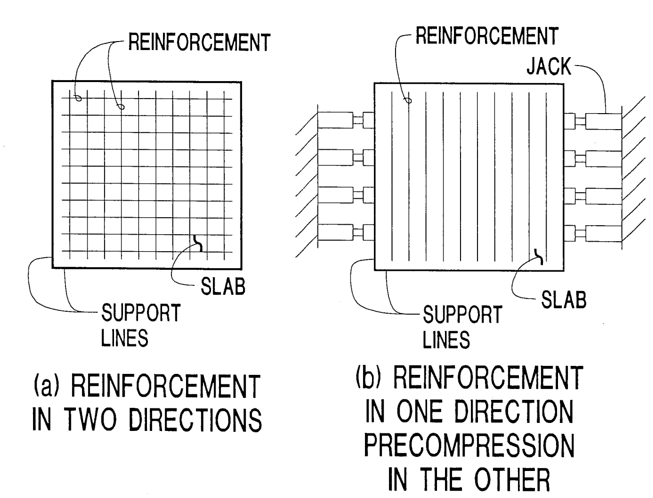


SIMPLY SUPPORTED SQUARE PLATE UNDER TRANSVERSE LOADING AND UNIAXIAL PRECOMPRESSION

FIGURE 2.2-4

Compare this slab with the example of Figure 2.2-2, where no precompression was provided, and where the slab was regarded as a two-way system for the magnitude of loading considered. In the example of Figure 2.2-4, the level of loading is increased to cause cracking of concrete in the y-direction. As a result, the initial two-way natural response of the slab of Figure 2.2-4 changes into a one-way system with increase in loading. From the foregoing, it is evident that the slab's response as a one- or two-way system can depend on the level of applied loading. This issue will be further discussed under the serviceability and limit state considerations.

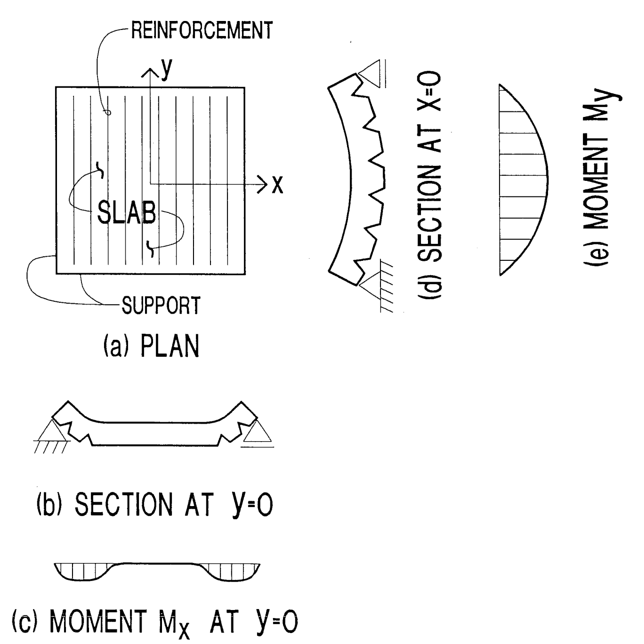
Nonprestressed reinforcement in two directions, as well as combinations of precompression and nonprestressed reinforcing can also result in two-way systems at service condition (Figure 2.2-5).



SIMPLY SUPPORTED SQUARE PLATES. NONPRESTRESSED (a), AND HYBRID (b) TWO-WAY SYSTEMS.

FIGURE 2.2-5

For completeness in treatment of the subject, review the deflection and load carrying characteristics of a slab supported on four sides and reinforced in one-direction. The deformation of the slab and its one-way load carrying response is illustrated in Figure 2.2-6 for the condition of slab subjected to uniform loading in excess of that needed to cause cracking of concrete.



SIMPLY SUPPORTED, UNIDIRECTIONALLY REINFORCED NONPRESTRESSED SLAB

FIGURE 2.2-6

A practical alternative for introducing precompression into a concrete slab is by way of post-tensioned tendons internal to the slab. Refer to Figure 2.2-7, where straight unbonded tendons anchored at the slab edges provide biaxial (Figure 2.2-7(a)), or uniaxial precompression (Figure 2.2-7(c)). The unbonded post-tensioned tendons are assumed to be straight and positioned at the mid-depth of the slab. The tendons are greased and contained in a bond-breaking sheathing. For the same amount of precompression, the in-service behavior of the two-way post-tensioned slabs of Figures 2.2-7(a) and 2.2-3 is the same. The latter has the precompression exerted through externally secured jacks.

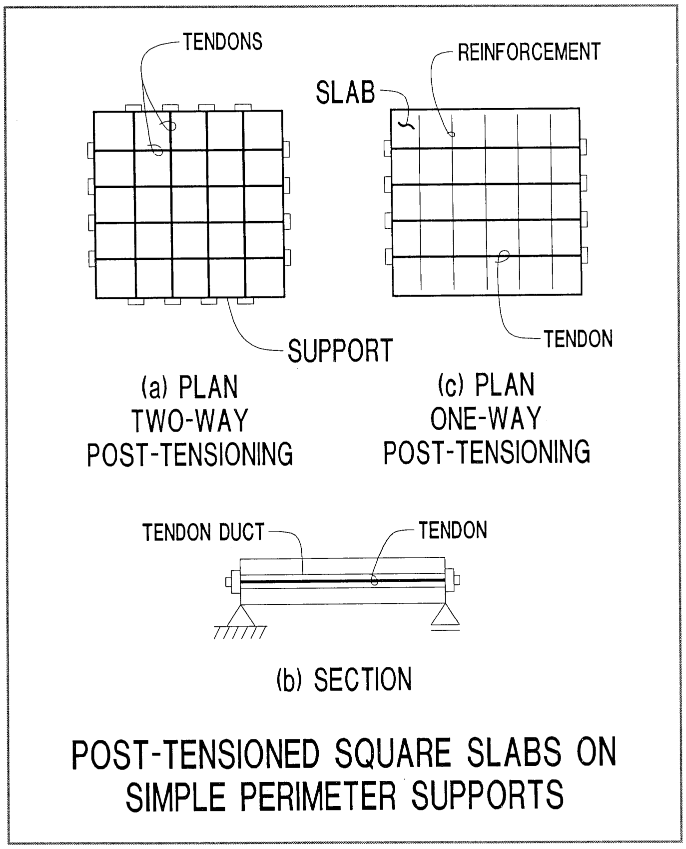


FIGURE 2.2-7

Post-tensioned tendons are not generally placed along a straight line, as adopted for the preceding example. The tendons are commonly profiled along a curve, such as to exert an uplift (lateral force) favorable to the slab response. The profiling and its associated uplift do not affect the present precompression discussion and the conclusions arrived at.

Also, in practice, internal tendons can be bonded to the concrete through grouting of the void of a tendon's duct, after completion of prestressing. Again, the grouting does not in principle affect the present discussion and its conclusions.

2.3 Definition of serviceability and strength limits

Because cracking and mobilization of reinforcement can change the load path, in the determination of the structural system, two states of the load path are of particular significance. One is the in-service condition which describes the response under working loads. The other is the strength limit state, when the safety of the structure against overload is established.

Serviceability refers to the function of the floor slab under its design-intended in-service condition. For prestressed slabs, the serviceability is controlled at the design stage through limits imposed on tensile stresses and deflections, both under working loads. Hypothetically computed and averaged tensile stresses are used as a guide to control cracking in slab. Deflections are limited to guarantee satisfactory slab function, as well as vibration control (Aalami 1989).

At **strength** limit state, the slab should develop adequate capacity to sustain an assumed overload. It is implemented to safeguard against the possibility of collapse and for life safety. The natural load path of a floor system in its in-service and strength limits may not necessarily be the same.

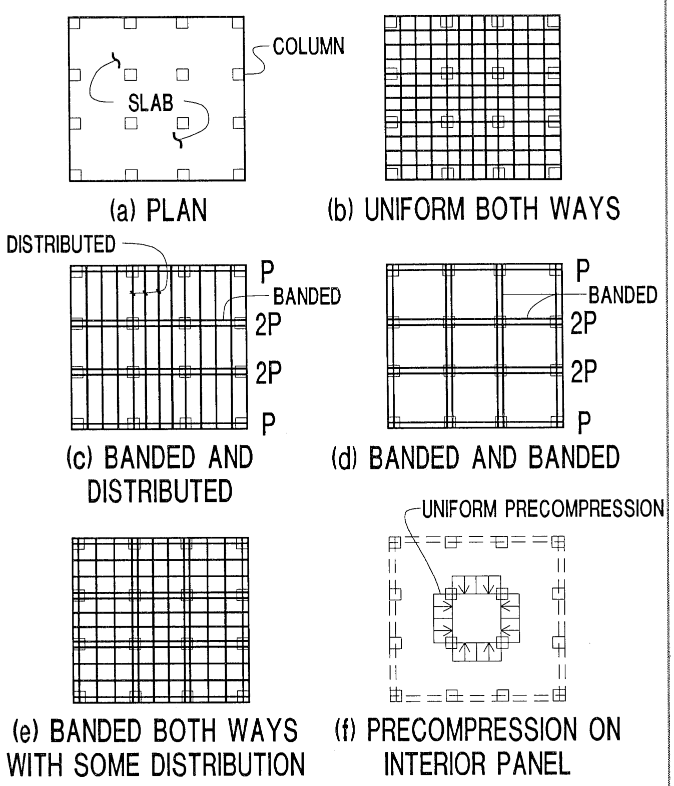
3—SERVICEABILITY

Serviceability check is achieved through tensile stress and deflection checks under working loads (ACI, 1992, UBC, 1991). While rigorous analytical capabilities for post-tensioned slabs are readily available, in everyday ACI-code-based design of post-tensioned slabs, both the stresses and deflections are computed under the assumption that the slab remains linearly elastic and uncracked. The slab's stiffness is based on its gross cross-sectional geometry. Moments and stresses are assumed smeared uniformly across the slab's tributary.

For serviceability design, the test of a two-way structural system is: (i) the availability of precompression in two perpendicular directions of a magnitude essentially equal to the average precompression used in overall design of slab (150 psi or more), and (ii) the development of curvature in both directions in the deflected slab.

Based on the foregoing test, the tendon arrangement of Figure 2.2-7(a) results in a two-way system, since it provides two-way precompression. The reinforcement arrangement of Figure 2.2-7(c) is viable and permissible, with the understanding that the system is one-way along the x-direction from the post-tensioning design standpoint. In the x-direction, the one-way minimum requirements of the code for prestressed slabs apply. Along the y-axis, no stress check needs to be performed, since the construction is viewed as nonprestressed in this direction. The minimum reinforcement and span to depth ratios in y-direction are governed by the non-prestressed sections of the code. As it will be discussed later, at the strength limit state, however, the arrangement is a two-way system for both configurations (parts (a) and (c)).

Consider a floor system supported on an orthogonal array of columns, such as shown in Figure 3-1. Let the post-tensioning in each direction consist of straight unbonded tendons positioned at mid-depth of slab; and let the prestressing amount to an average precompression of 150 psi. Four different arrangements of prestressing tendons are illustrated. The compression delivered at the boundary of the floor slab along its centroidal axis will diffuse readily into a uniform precompression in regions away from the slab edges. As a result, the four arrangements shown lead to a practically uniform precompression for the interior panel shown in Figure 3-1(f). In other words, from a serviceability standpoint, for all the arrangements shown, the interior panels respond in a two-way manner.



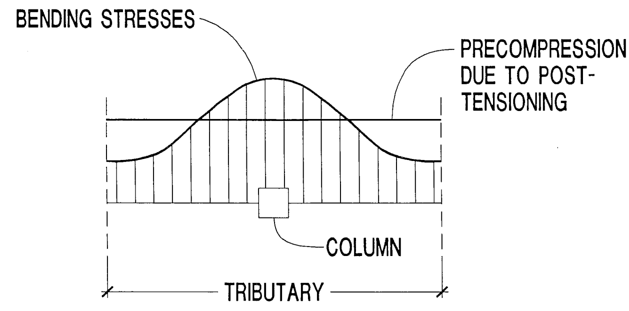
PRINCIPAL ARRANGEMENT ALTERNATIVES FOR PRESTRESSING TENDONS

FIGURE 3-1

For straight and unbonded tendons, placed at mid-depth of the plate, there is no practical difference among the different options shown, when considering the serviceability of the interior spans. However, if the tendons are profiled, in addition to precompression, each tendon exerts an uplift along its length. The uplift of the profiled tendons causes bending in slab. The bending, by design counteracts that of the gravity loading.

For all the cases shown, an applied loading, such as live loading, would cause the same bending stresses. The bending stresses generated by nonprestressed loading are not a function of tendon arrangement.

Refer to Figure 3-2. Observe that the distribution of stress in a slab, such as along a column line, is generally nonuniform with a pronounced peak over the column supports. But the precompression due to prestressing is essentially uniform in most designs.



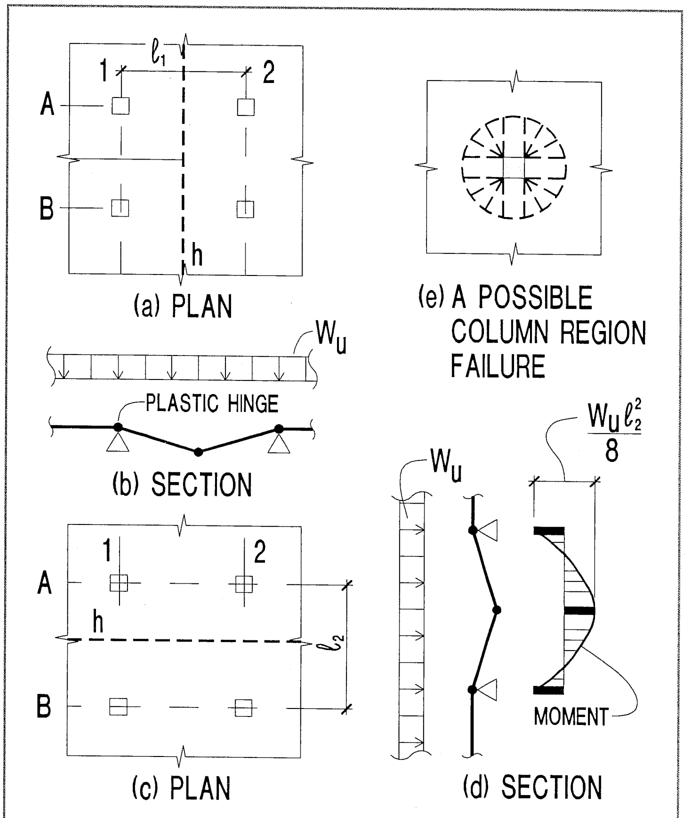
DISTRIBUTION OF BENDING STRESSES AND PRECOMPRESSION DUE TO POST-TENSIONING

FIGURE 3-2

4—STRENGTH LIMIT STATE

The strength limit state ensures that the structure has capacity reserve equal or exceeding the overload level stipulated in code. One method of evaluating a floor system's strength limit is to magnify the elastically computed moments by the overload factor, and to verify that the sections along each member can withstand the magnified actions. This is the method most common in North America. An alternative is to envisage failure modes (collapse mechanism) for the structure, and re-compute the actions based on the assumed collapse mechanisms. The mechanism for which the member sections at all locations can withstand the respective sectional actions is the acceptable solution.

Consider the partial plan of a flat plate floor system shown in Figure 4-1. On the assumption that the column size and column/slab region is adequate to avoid a preemptive punching failure similar to the mechanism illustrated in Figure 4-1(e), the floor system is likely to fail in one of the two modes shown in parts (a) and (c) of the figure. Observe the first alternative (parts, a, and, b), where plastic hinge lines are formed at grid lines, 1, 2, and, h. In order for the mechanism shown to develop, the entire reinforcement along the grid lines, 1, 2, and, h, would be mobilized, since the hinge lines extend over the entire length of the slab. The entire load, Wu, (Figure 4-1(b)) would transfer to support lines 1 and 2. Refer also to parts (c) and (d) of the figure, where the hinge lines A, B and h resist the entire loading, Wu, and consequently its static moment in the central panel ($W_uL^2/8$). It is apparent that for the mechanisms shown, at strength limit state, the entire loading must be considered to be resisted once in one direction, and independently in the other direction. The distribution of reinforcement along the line of hinges is not as critical, as is its amount and its position within the depth of the section. Clearly along lines, 1, and, 2, of Figure (b) the reinforcement/prestressing must be at the top, while along line, h, it must be at the bottom to be most effective.



SIMPLIFIED FAILURE MECHANISMS OF FLAT PLATES

FIGURE 4-1

Based on the foregoing, the four reinforcement and prestressing arrangements shown in Figure 3-1 all develop a two-way system for the strength limit analysis. Also, both slabs of Figure 2.2-7 are two-way systems in their strength limit response, although the slab in part (c) of the figure was considered a hybrid of one-way systems for serviceability consideration.

The bi-directionally reinforced slab in Figure 2.2-5(a) is a two-way system, while the slab in part (b) of the same figure is a one-way system in direction of nonprestressed reinforcement at strength limit state. This slab can not develop a viable failure mechanism in direction of applied precompression. As loading is increased, the pre-cracking two-way path of load transfer would transform to a one-way load path in direction of non-prestressed reinforcement when slab approaches its strength limit.

When evaluating a structure for strength limit state, it is important to differentiate between the strength *demand* and the strength *limit.* Refer to Figure 2.2-3, where a plain concrete slab is subjected to biaxial precompression. It is stated in the forgoing, that the slab can be serviceable, but would not develop a collapse mechanism, based on which a strength limit could be assessed. Notwithstanding, if the factored (magnified) values of actions for the strength limit state are too small to cause cracking in the slab, the slab would meet the *demand* on strength limit. It is argued that for the small loading imposed, the slab is a two-way system for both the in-service and the strength limit conditions. In other words, the level of strength demand is a factor in determination of load path and the load resisting capability of a slab. In practice, however, when designing new construction, the strength limit state is normally tuned to be close to the strength demand for economy of construction. It is in the *evaluation* of existing structures, when an issue of the nature discussed can arise.

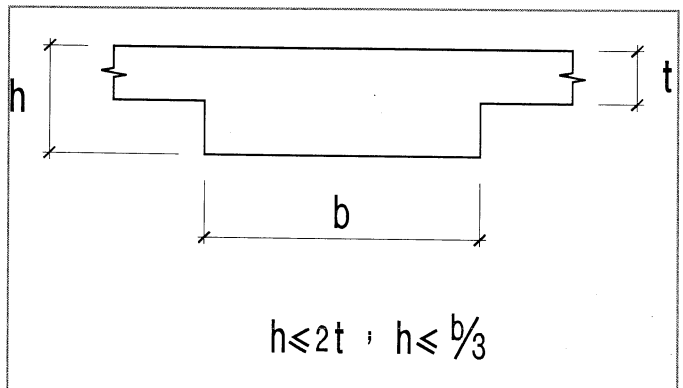
5—CODE APPLICATION AND DISCUSSION

The discussion of code application is focused on ACI (ACI-318, 1992, ACI-423, 1989) and Uniform Building Code (UBC, 1991). The code views the slab system at two states, serviceability limit and strength limit. The qualification of a prestressed floor as one- or two-way system governs the amount of the code required minimum reinforcement, as well a member's strength requirement.

In the interpretation of the code requirements, it is important to be clear about the definition of a beam in prestressed floor systems.

5.1 Beam Definition for Prestressed Floors

A beam is defined as a thickening in slab which (i) exceeds the limitations set forth in Figure 5.1-1, (ii) extends between two slab supports, such as columns, and, (iii) provides support for the slab through the flexure of the thickened region. In post-tensioned slabs, it becomes necessary at times, to thicken a region of the slab for the purpose of providing increased tendon drape and tendon cover. The primary support to the slab loading is provided by the uplift force of the draped tendons in the thickened region, as opposed to the increased local stiffness which is the case in beams with heavy and deep stems. The slab-beam, or the wide-shallow-beam— as it is referred to at times, and as illustrated in Figure 5.4-1 is not stiff enough to significantly influence the stiffness characteristics of a floor system due to added loading (live loading). Further, since the region is treated as part of the slab entity, no beam-dedicated stirrups are provided in such a thickening for one-way shear . The design is governed by the geometry and the structural system of the slab, of which the thickening is an appendage.



CROSS-SECTION OF A SLAB-BEAM

FIGURE 5.1-1

5.2 Simple Beam/Slab Construction

In typical beam/slab constructions (Figure 5.2-1), the beam span is between three to four times the slab span. The reinforcement in the slab is primarily unidirectional— perpendicular to the beam. Although in the common case the slab is under biaxial precompression, by virtue of its predominant loading and geometry, the response of the slab is essentially a cylindrical deformation (Figure 2.2-1(b)). As a result, the slab is considered a one-way system. The precompression in direction perpendicular to the slab span may be used for control of cracking due to shrinkage and temperature, as well as spreading of concentrated loads. The beam too, is a one-way system. The beam is assigned a load equal to its tributary, but designed in bending for an effective width smaller or equal to the tributary (Aalami 1993b)

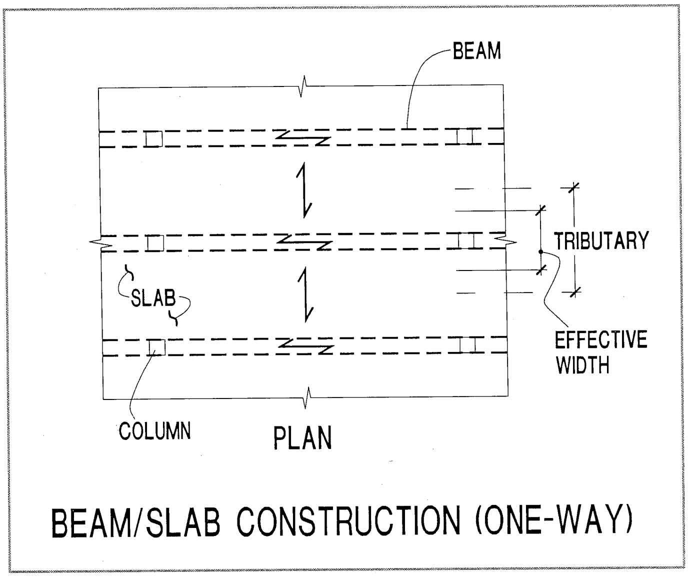


FIGURE 5.2-1

5.3 Flat Slab on Intersecting Beams

A formerly common system of construction for nonprestressed floors was the uniform slab on an orthogonal grid of beams and columns as shown in Figure 5.3-1. This method of construction—now less common in nonprestressed concrete design—is generally not used in post-tensioned construction. It is included in this review for completeness.

Regardless of the arrangement of prestressing, as long as the average precompression in two-directions is essentially equal to the design value, the slab would respond in a two-way manner for service condition. Hence, the two-way system of minimum reinforcement of the code for the prestressed slabs applies.

For strength limit state, the two commonly used options are as indicated in parts (b), and (c) of Figure 5.3-1. Using a *simple frame*, as illustrated in part (b), the slab is designed as an independent structural member—acting as a two-way system. Beams are designed separate from the slabs. For example, the beam marked on line A, is designed for a loading associated with the hatched tributary. In other words, the weight of the slab is broken into four parts, each carried by one of the beams at its perimeter. The beams are designed as a one-way system subjected to the triangular loading (or trapezoidal, depending on the geometry of the arrangement). The beams are designed with an effective width equal to, or smaller than, the maximum load tributary. For this case, the code's one-way minimum reinforcement and provisions apply.

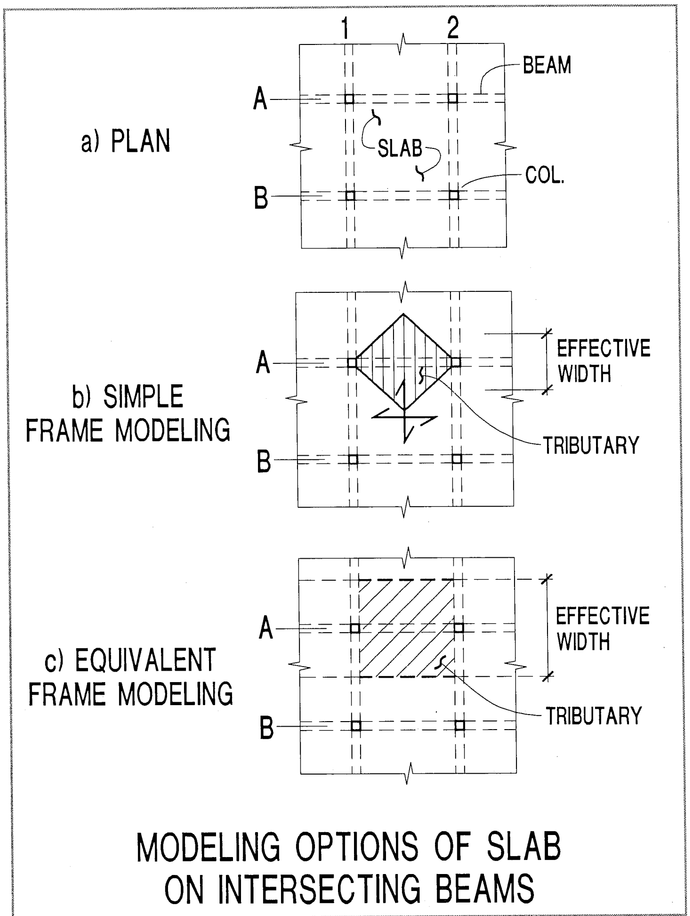


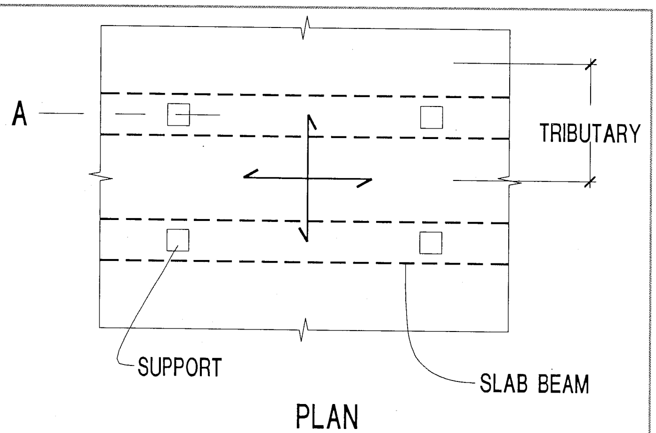
FIGURE 5.3-1

In the *Equivalent Frame Modeling* (Chapter 13 of ACI, 1992) method, each beam, such as the one marked on line A, is considered in combination with its slab tributary. The slab tributary extends to mid-distance between column lines transverse to the frame being designed. The loading on the frame is represented by the hatched area in the figure. The beam-slab combination and the loading shown are designed with an effective width equal to, or greater than, the entire tributary (Aalami, 1993b). For the equivalent frame modeling, the code's two-way minimum reinforcement applies to the beam-slab combination. Note that in this scheme the beam and slab are not designed as two independent members, as is the case in the simple frame.

It is interesting to note that the equivalent frame modeling used in part (c) of the figure results in a larger total bending moment for the beam/slab combination, than the beam of the simple frame of part (b). The simple frame has a smaller total static moment—on account of a smaller loading assigned to it. But, since it is a one-way system, it is subject to a more stringent design requirement when using UBC. UBC requires that one-way systems using unbonded post-tensioning be designed to withstand their selfweight and 25% of the unreduced live loading by means other than unbonded post-tensioning. According to UBC, the simple frame modeling would generally result in less post-tensioning, but more nonprestressed reinforcement. ACI does not follow the UBC's stringent reinforcement requirement for one-way systems when using unbonded tendons.

5.4 Wide shallow beams

An economical design solution for post-tensioned slabs on column grids with aspect ratios between 1.5 and 2.5 is the use of wide shallow beams (Figure 5.4-1). A wide shallow beam is a thickening of slab in the long direction between the column supports with dimensional restrictions given in Figure 5.1-1. The wide shallow beam system allows the slab thickness to be based on the dimensions of the shorter span. A wide shallow beam slab construction is a two-way system for both the serviceability and strength limits of design. It is designed using the Equivalent Frame Method for modeling of two-way systems.



SLAB-BEAM (WIDE SHALLOW BEAM) CONSTRUCTION

FIGURE 5.4-1

6— OTHER CONSIDERATIONS

6.1 Profiled Tendons

Profiled tendons are placed on a path other than straight. Profiled tendons exert to the containing member, forces lateral (perpendicular) to the tendon, and longitudinal (in direction of tendon) along the tendon path. Generally, the profile is selected such as to obtain a favorable distribution of lateral tendon force (uplift). A detailed treatment of forces exerted by a tendon on its supporting structure is given in references (Aalami 1990, Aalami, 1993a, Collins and Mitchell, 1991). Apart from location of abrupt changes in tendon angle, the longitudinal force is primarily due to friction of tendon with its sheathing. The inclusion of tendon friction affects some of the stress distributions discussed in the forgoing. However, the nature of its impact does not alter the concepts presented and the conclusions arrived at for the one-way and two-way system categorization.

6.2 Stiffened Slabs

Slabs may be stiffened in one- or two-directions. The slabs stiffened in one-direction are referred to as ribbed- or joist-slab construction. Those stiffened in two-directions are called waffle slabs (also referred to as coffered slabs)

6.2.1 JOISTS
Joist slabs have a higher stiffness in the joist direction than the perpendicular direction. The conclusions arrived for slabs of uniform thickness do not directly apply to joist slabs. Due to its higher stiffness, a smaller curvature along the joist carries a larger moment than the same curvature in perpendicular direction. The aspect ratio, for which a ribbed slab may—on the basis of its curvature response—be considered to be acting as a two-way system depends on the geometry of the construction.

6.2.2 WAFFLE
Stiffeners in waffle slab construction are generally spaced uniformly in both directions. This results in uniform stiffness of the floor in two directions. Waffle slabs can, therefore, be treated as a uniform slab with an equivalent thickness. Solid waffles can be treated as column caps and drops when comparing the response of a waffle construction with a conventional slab. Modeling of waffle slabs is discussed in reference (Aalami, 1989). Waffle slabs are viewed as slabs with uniform thickness with respect to their response as one-or two-way structural systems.

6.3 Grouted Tendons

There is no basic difference between grouted and unbonded tendons in regards to the one- or two-way action of a floor system. The friction loss is more pronounced in grouted tendons than the unbonded alternative, but not to the extent that would invalidate the concepts and conclusions offered.

7—CONCLUDING REMARKS

The considerations necessary to determine whether a floor system acts as a one- or two-way system are presented and discussed. The following conclusions are made.

In most cases the structural modeling for a concrete floor system is not unique. Distinction is made between the structural system associated with the floor's natural response to the applied loading, the system adopted by the engineer for design of the floor, the system selected for verification of floor's adequacy, the system under working conditions, and finally the strength limit system.

The structural system for in-service response may be different to that of strength limit state. A slab which responds as a one-way system in-service, may develop a two-way system at its strength limit state.

For serviceability consideration of a post-tensioned structure, the precompression together with the development of *curvature* in direction of *precompression* under applied loading are the tests for mobilization of a load path. If the precompression and curvature tests pass for two directions, the floor is a two-way system, otherwise it is a one-way.

The *strength demand* is defined as the resistance a structure must develop when subject to its design overload. The capacity of a structure can be more. The load path developed at strength demand might be different from that of the slab's load path at capacity. The load path associated to strength demand determines the categorization of a slab into a one- or two-way system.

Geometry of a floor system by itself is not adequate to identify the structural model of a floor system. The level of loading, disposition of prestressing and reinforcement, as well as the modeling envisaged by the design engineer are all factors governing the definition of the structural model.

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APPENDIX A

A.1 Moment displacement relationships

The relationship between the moments in a plate on x-y plane, and the displacements, w, perpendicular to the plane of the plate are (Timoshenko et al, 1959, Aalami, 1977):

$$M_x = -D(w_{,xx} + \nu\, w_{,yy})$$

$$M_y = -D(w_{,yy} + \nu\, w_{,xx})$$

$$M_{xy} = D(1-\nu)w_{,xy}$$

Where,

a comma followed by a subscript represents partial differentiation in turn with respect to each subscript variable;

ν = Poisson's ratio

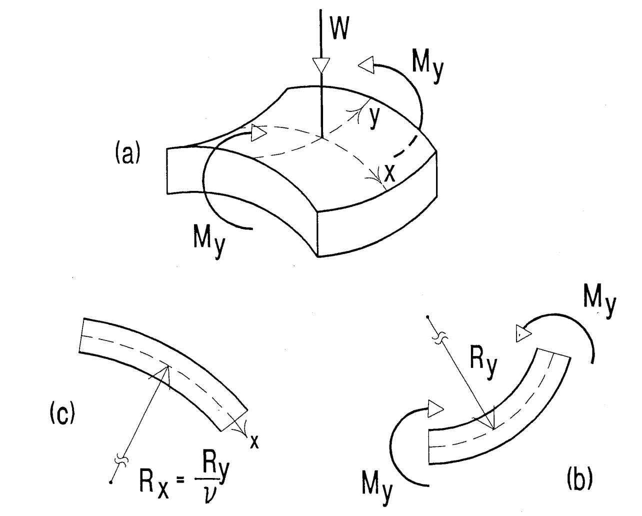
D = unit stiffness [$h^3/(1-\nu^2)$, h is plate thickness]

A.2 Anticlastic deformation

A plate element bent in one direction, such as y-direction shown in Figure A-1, develops a curvature in the perpendicular direction (x-direction in the figure), due to Poisson's ratio (ν). The relationship between the curvature in direction of applied moment ($1/R_y$) and the transverse direction is:

$$1/R_x = \nu/R_y$$

The curvature developed in the transverse direction deforms the plate into a saddle shape. It is called anticlastic curvature.



TWO-WAY CURVATURE (ANTICLASTIC) OF PLATE ELEMENT UNDER MOMENT IN ONE-DIRECTION

FIGURE A-1

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