Hyperstatic (Secondary) Actions In Prestressing and Their Computation

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SYNOPSIS

This Technical Note describes the definition, computation, and the significance of hyperstatic (secondary) actions in the design of pre-stressed concrete structures. The Note covers the cases of (i) skeletal structures, such as beams and slabs modeled as beam frames, (ii) continuums, such as plates, shells and solids, and (iii) segmental construction, such as prestressed and post-tensioned spliced girders. Each case is illustrated through a numerical example.

1 - INTRODUCTION AND DEFINITION

1.1 Definition of Hyperstatic (Secondary) Actions

Hyperstatic (or secondary) actions develop in a prestressed member due to prestressing forces, and as a consequence of the constraint by the supports to the free movement of the prestressed member. If a prestressed member is allowed to displace freely, as in the case of determinate structures or precast members prior to alignment and installation, no hyperstatic (secondary) moments are generated. However, in most cast-in-place construction, where supports constrain movement of the prestressed member, hyperstatic actions can be significant and therefore must be accounted for in a design.

1.2 Hyperstatic Actions and Sectional Strength

Consider the hypothetical case of a two-span, post-tensioned beam, which was cast and stressed prior to installation (Fig. 1.2-1). To eliminate the impact of self-weight from the discussion, assume that the beam is resting freely on its side. Before installation, the tendon forces cause the beam camber indicated by the curved soffit in Fig. 1.2-1(b). The camber is due solely to the flexing of the beam under the action of its prestressing tendons. At installation, the beam must be forced down at the center to become straight before it can be tied to the aligned supports shown in Fig. 1.2-1(a). The forces at the supports necessary to hold the beam in its designated position are indicative of hyperstatic (secondary) reactions that would have been generated if the beam had been cast first, and stressed afterwards. Fig. 1.2-2(a) shows the hyperstatic reactions of the beam held on simple supports.
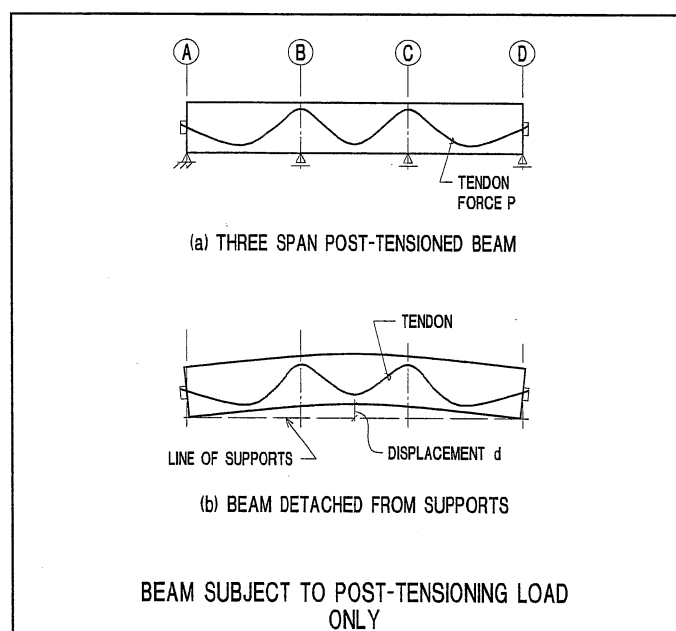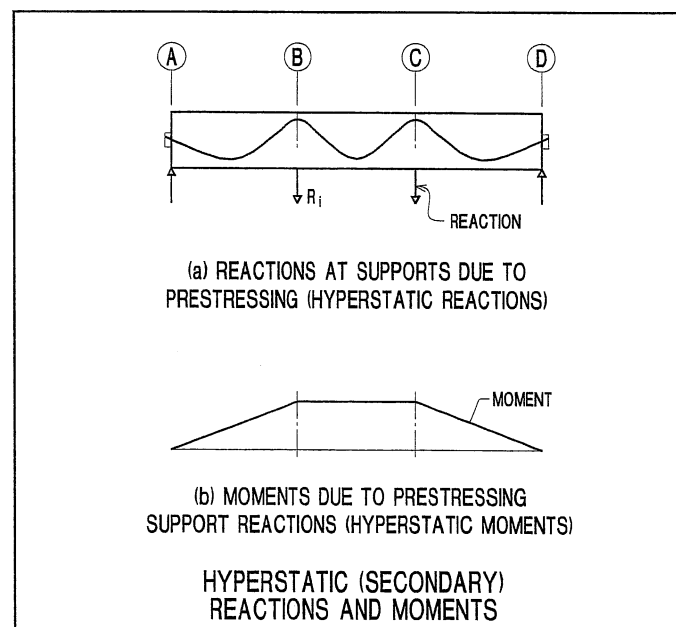
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(a) THREE SPAN POST-TENSIONED BEAM

(b) BEAM DETACHED FROM SUPPORTS

BEAM SUBJECT TO POST-TENSIONING LOAD ONLY

FIGURE 1.2-1

(a) REACTIONS AT SUPPORTS DUE TO PRESTRESSING (HYPERSTATIC REACTIONS)

(b) MOMENTS DUE TO PRESTRESSING SUPPORT REACTIONS (HYPERSTATIC MOMENTS)

HYPERSTATIC (SECONDARY) REACTIONS AND MOMENTS

FIGURE 1.2-2

In cast-in-place members, the sequence is reversed. Now assume the beam is cast while positioned on the supports and then the tendons are stressed afterwards (Fig. 1.2-1(a)). When the tendons are stressed the camber shown in Fig. 1.2-1(b) is prevented and additional reactions are developed at the supports. These reactions are called the hyperstatic (secondary) actions. The reactions at the supports are due to post-tensioning only (self-weight is treated separately). The free-body diagram of the beam detached from its supports is shown in Fig. 1.2-2(a). Since, in this example, the supports are assumed hinged, no moments are developed at these locations. These reactions, being the only forces in the beam, must form a self-equilibrating system. That is to say, the sum of hyperstatic (secondary) reactions must be zero:

$$S \ R_{hyp} \ = 0 \qquad (1)$$
$$S \ M_{hyp} \ = 0 \qquad (2)$$

where R_{hyp} is the hyperstatic reaction and M_{hyp} is the hyperstatic moment. In this example, there are no hyperstatic moment reactions at the supports. Fig. 1.2-2(b) illustrates the distribution of hyperstatic (secondary) moments in the beam.

Pursuing the same example, in which only the post-tensioning forces are being reviewed, observe in Fig. 1.2-3(a) that at any section along the beam, the hyperstatic reactions induce a hyperstatic moment and a hyperstatic shear. There is no resultant horizontal force at the cut section for the roller-support example considered. From the statics of the free-body diagram of the cut beam, the hyperstatic moment and shear are given by the following relationships

$$V_{hyp} \quad = S \ R_i \qquad (3)$$
$$M_{hyp} \quad = S \ R_i * X_i \qquad (4)$$

(a) RESULTANT MOMENT AND SHEAR AT A SECTION

(b) ASSUMED INTERNAL FORCE DISTRIBUTION AT STRENGTH LIMIT STATE

SECTIONAL ACTIONS DUE TO PRESTRESSING & THE CORRESPONDING INTERNAL DISTRIBUTION OF FORCES AT STRENGTH LIMIT STATE
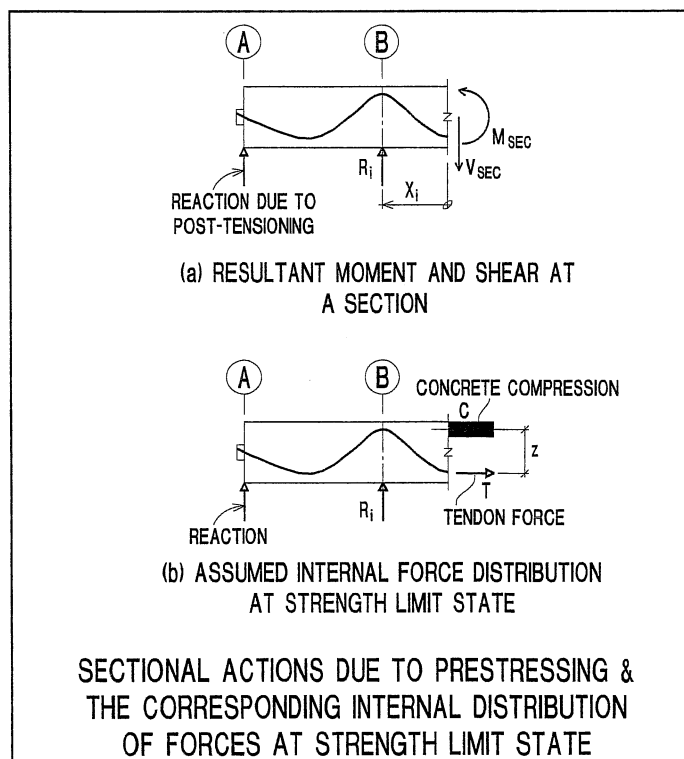
FIGURE 1.2-3

The hyperstatic shear and moment shown in Fig. 1.2-3(a) at the cut section are sustained by forces developed in the concrete and reinforcement over the cross section. At the strength limit state, the moment is assumed to be resisted by a compression block and a tensile force as shown in Fig. 1.2-3(b), from which the following relationships apply:

$$C \quad = T \qquad (5)$$
$$M_{hyp} \ = Tz = Cz \qquad (6)$$

where,

C = total compression force;
T = combined tension force due to the prestressing and nonprestressed reinforcement;
z = internal lever arm of the section.

From the foregoing, it is evident that at the strength limit state, the section must develop an internal resistance associated with the hyperstatic moment. This resistance is in addition to that required by the action of other loads, such as self-weight and live loading. ACI [ACI-318, 1995], and AASHTO [AASHTO, 1994] require that the hyperstatic (secondary) actions be included with a factor of 1.0 in all strength demand load combinations, when actions are calculated using elastic theory. For example, one of the strength load combinations stipulated in the ACI code for gravity conditions is

$$M_n = \quad \frac{(1.4M_d + 1.7M_\ell + M_{hyp})}{f} \qquad (7)$$

where,

M_n = nominal strength of section;
f = strength reduction factor;
M_d = moment due to dead loading; and
M_ℓ = moment due to live loading.

The hyperstatic moment is not factored for several reasons. First, the parameters governing its value are more accurately known than those associated with the dead and live loads. Second, in most cases, hyperstatic moments counteract the moments due to dead and live loading. Hence, an increase in the load factor for the hyperstatic moments is often not conservative.

The preceding discussion demonstrates that, for strength considerations, the hyperstatic moments enter the computations, and not the primary or balanced moments.

2 - COMPUTATIONAL PROCEDURES

Hyperstatic moments can be calculated either directly or indirectly. For skeletal members, such as beams, and floor systems that are modeled as strips of isolated slab frames, hyperstatic actions can be successfully calculated using both methods. For continuum members, such as floor slabs that are viewed as plates, the direct method must be

used. The background to these methods is explained in [Aalami, 1990]. Both methods are outlined below, in the context of both skeletal and continuum structural models. Later the special requirements for the analysis of segmental construction are addressed.

2.1 Skeletal Structures

2.1.1 Direct Method

The direct method is based on the definition of hyperstatic actions (moments and shears) as described in the preceding section. Observe a typical frame as shown in Fig. 2.1.1-1. The frame is subject to dead loading, live loading and post-tensioning. The reactions caused at the supports are due to all the loading components - namely, dead, live and post-tensioning or prestressing. Fig. 2.1.1-2 illustrates the free body diagram of the slab/beam member in which only the post-tensioning and its reactions at the supports are shown. The free body diagram is complete as shown in the figure (note that there is no dead or live loading shown). It is not necessary to include the balanced loading due to post-tensioning in this diagram, since the tendon is considered in-place in the loading diagram. Note, that the balanced loading appears in the loading diagram only if the tendon is assumed removed from its housing.
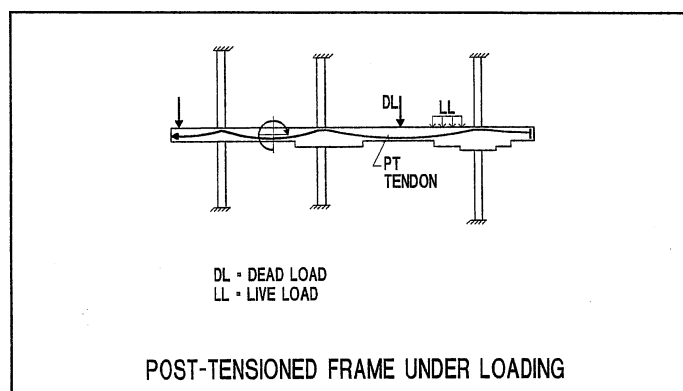

DL - DEAD LOAD
LL - LIVE LOAD

POST-TENSIONED FRAME UNDER LOADING

FIGURE 2.1.1-1

By definition, the moments and reactions shown in Fig. 2.1.1-2 are the hyperstatic actions, since these are the actions induced by post-tensioning alone. For static equilibrium, the sum of all the hyperstatic actions on the frame must add to zero.
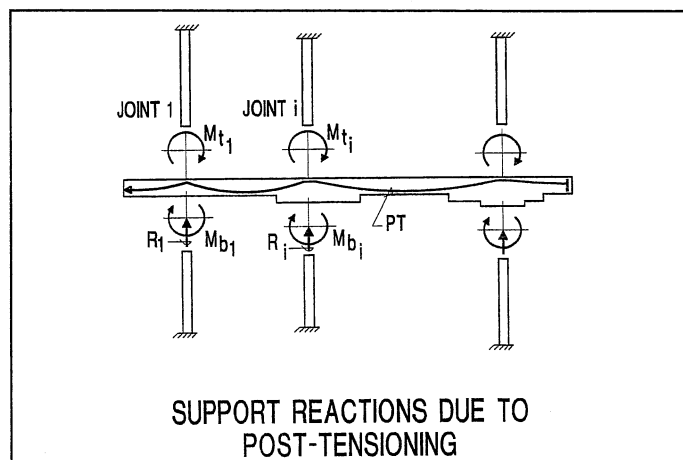

SUPPORT REACTIONS DUE TO POST-TENSIONING

FIGURE 2.1.1-2

A common procedure for the direct method is that the prestressing forces are first translated into the corresponding balanced loading to be applied to the frame. The hyperstatic reactions at the supports are then computed by solving the frame for the balanced loading due to post-tensioning/ prestressing [ADAPT, 1997]. It is clear that the central issue in the correct computation of hyperstatic reactions is the faithful representation of the balanced loading as a set of external actions applied to the frame. Only if all the components of the balanced loading are considered, will the calculated hyperstatic actions be correct.

The hyperstatic reactions calculated above are then applied to the member to determine the hyperstatic moment and shear at any point along its length (Fig. 2.1.1-3). At any distance X_i, as shown in Fig. 2.1.1-4, the hyperstatic shear is simply the algebraic sum of all reactions, and the hyperstatic moment is the moment of all actions. The relationships are as follows:

$$V_{hyp} \quad = \quad S\, R_i \tag{8}$$

$$M_{hyp} \quad = \quad S\,(M_{t_i} + M_{b_i}) + S\,(R_i * X_i) \tag{9}$$

Where,

M_{t_i}, M_{b_i} and R_i	=	Support reactions due to post-tensioning;
X_i	=	Distance to the section under consideration.

ACTIONS CAUSING HYPERSTATIC MOMENTS AND SHEARS

FIGURE 2.1.1-3

HYPERSTATIC MOMENT (M) AND
SHEAR (V) IN A SECTION

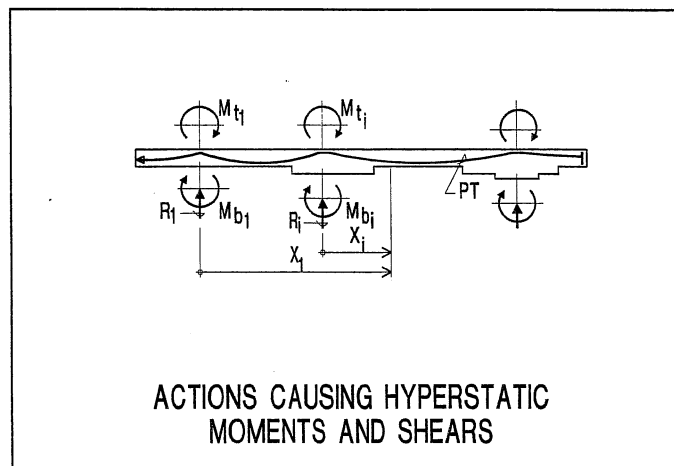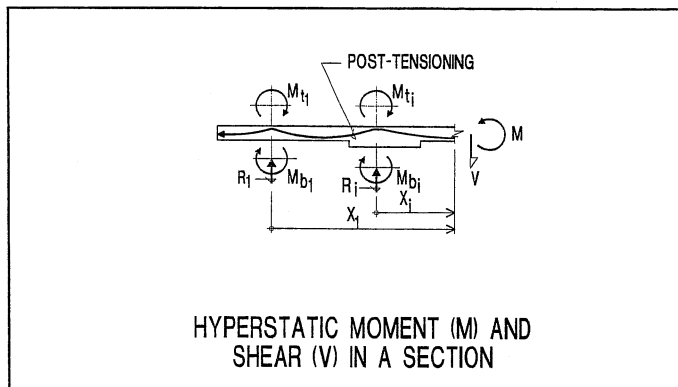**FIGURE 2.1.1-4**

2.1.2 Indirect method

The indirect method is a procedure commonly used for the calculation of hyperstatic moments in skeletal structures. It is based on the following relationship [ADAPT, 1997]:

$$M_{hyp} = M_{bal} - P*e \qquad (10)$$

Where,

e = eccentricity of post-tensioning/prestressing with respect to the neutral axis of the section (positive if CGS is above the neutral axis, otherwise negative);

M_{hyp} = hyperstatic moment;

M_{bal} = balanced moment due to balanced loading;

P = post-tensioning/prestressing force (positive).

In this relationship moments causing tension at the bottom fiber are assumed positive. The hyperstatic reactions and shears are then calculated from the hyperstatic moments.

2.1.3 Comparison of the two methods

Both methods yield the same results, provided the balanced loading is correctly modeled.. Otherwise, both solutions are approximate, with the difference that the direct method's solutions are more consistent in satisfying the overall equilibrium of the structure.

Apart from its generality and accuracy, an advantage of the direct method is that it does not require the knowledge of the eccentricity of the tendon (e), or its force (P) for the computation of hyperstatic moments, once the post-tensioning reactions are computed.

2.1.4 Example of a Skeletal Structure

Consider a design strip from a two-way floor system, which is being designed using a slab frame modeling technique, such as the Equivalent Frame Method (refer to Fig. 2.1.4-1). A three-dimensional view of the frame is shown in Fig. 2.1.4-2. Herein, first the direct method is used to compute the hyperstatic moments of the two-way slab example. The hyperstatic moments obtained are checked against the conventional method (indirect method) of calculation to demon-

strate that both schemes yield the same solution. For this example the design strip was analyzed using the ADAPT-PT software package. The software accurately captures the balanced loading in the structural model. The software directly employs the definition of hyperstatic actions (moments and shears), as stated in the preceding, for the computation of hyperstatic moments (the direct method).
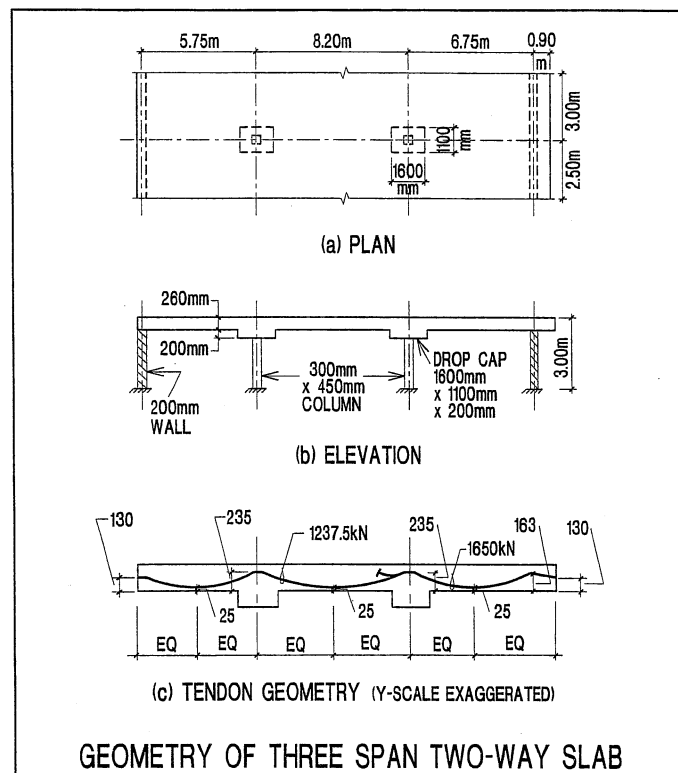

GEOMETRY OF THREE SPAN TWO-WAY SLAB

FIGURE 2.1.4-1
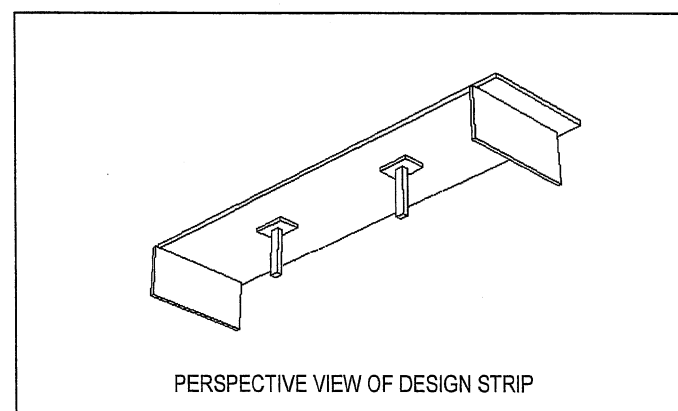

PERSPECTIVE VIEW OF DESIGN STRIP

FIGURE 2.1.4-2

The hyperstatic actions at the supports of the two-way slab example are shown in Fig. 2.1.4-3(a). Note that the sum of all hyperstatic reactions is zero.

S Reactions = 8.983 - 8.972 - 9.015 + 9.005 = 0.001 kN

(a) HYPERSTATIC ACTIONS AT SUPPORTS
(UNITS IN kN AND kN-m)

(b) DISTRIBUTION OF HYPERSTATIC MOMENTS
(UNITS kN-m)

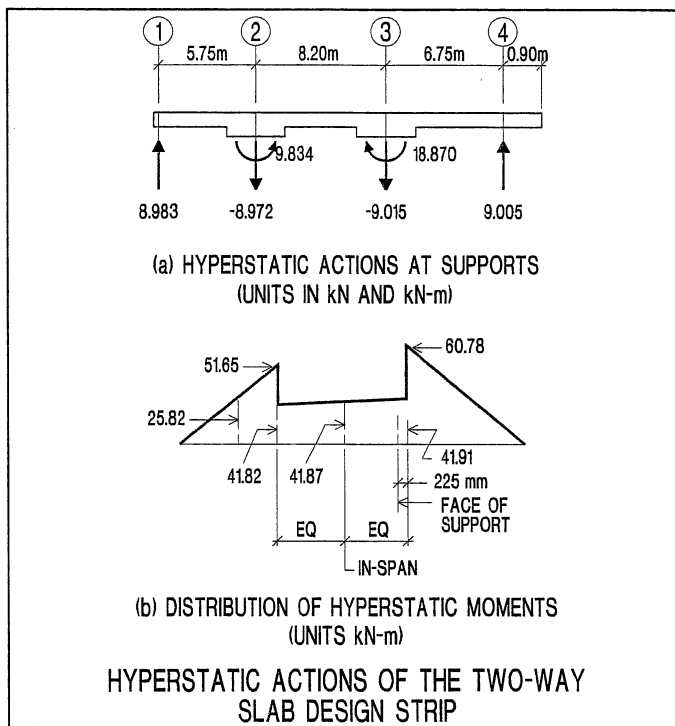**HYPERSTATIC ACTIONS OF THE TWO-WAY
SLAB DESIGN STRIP**

FIGURE 2.1.4-3

The hyperstatic actions constitute a self-equilibrating force system. Also note in this case that the third span has a different post-tensioning force than the first two spans. Added tendons extend from the tip of the cantilever at right to one-fifth point of the second span from the third support. The force in the added tendons is:

$$1650 - 1237.5 = 412.5 \text{ kN}$$

The solution for hyperstatic actions in this case is dependent on the accurate representation of the balanced loading of all tendons, including the portion of the added tendons from the third support to where they are dead ended in span 2. Ignoring the added effects of these tendons, including the portion in span 2, would adversely affect the accuracy of the solution (see Fig. 2.1.4-4 for the distribution of balanced moments in the model).
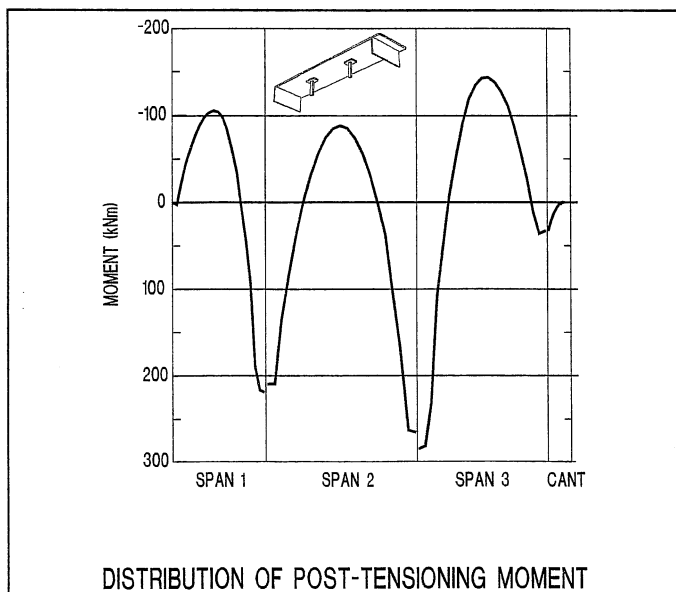

DISTRIBUTION OF POST-TENSIONING MOMENT

FIGURE 2.1.4-4

A - Direct Method:

The hyperstatic moments in the frame are determined using the hyperstatic actions at the supports and the statics of the structure.

Moment at left of support 2:

$$M_{hyp} = 8.983 \text{ kN} * 5.75 \text{ m} = 51.652 \text{ kNm}$$

At right of support 2:

$$M_{hyp} = 51.652 - 9.834 = 41.818 \text{ kNm}$$

At mid-length of span 1:

$$M_{hyp} = 8.983 * 2.875 \text{ m} = 25.82 \text{ kNm}$$

Moments at other locations are calculated in a similar manner. The complete distribution of hyperstatic moments is shown in Fig. 2.1.4-3(b).

B - Indirect Method (Conventional Method):

Using the conventional (indirect) method, the hyperstatic moments at the locations specified above are recalculated as follows:

$$M_{hyp} = M_{bal} - P * e$$

The distribution of balanced moments (M_{bal}) is shown in Fig. 2.1.4-4. The section centroid is located 130 mm above the slab soffit at midspan; and is computed to be 299.37 mm above the drop cap soffit at the centerline of supports 2 and 3.

At the center of span 1:

$M_{bal} = -104.10 \text{ kNm}$ (see Fig. 2.1.4-3)
$P = 1237.50 \text{ kN}$
$e = 25 - 130 = -105 \text{ mm}$

Note that eccentricity above neutral axis is taken as positive.

$$M_{hyp} = -104.10 + 1237.50 * 105/1000$$
$$= 25.83 \text{ kNm} \quad (= 25.82 \text{ kNm from Direct Method})$$
OK

At the left of the third support:

$M_{bal} = 265.75 \text{ kNm}$
$P = 1650 \text{ kN}$
$e = 235 + 200 - 299.33 = 135.67 \text{ mm}$
$M_{hyp} = 265.75 - 1650 * 135.67/1000$
$= -41.89 \text{ kNm} \quad (= 41.82 \text{ kNm from Direct Method})$
OK

Note that the results from both the direct and indirect methods are virtually identical.

2.2 Continuums (plates, shells and solids)

2.2.1 The Direct Method

The calculation of hyperstatic actions in continuum structures, such as slabs, shells and solids, presents a special challenge. The actions generated in a continuum structure due to the constraint of the supports distribute in two or three dimensions, in a manner that can not readily be related to the position and force magnitude of individual tendons. Since the one-to-one correlation that exists between a tendon in a skeletal structure and the associated post-tensioning moment is not apparent in a continuum, the indirect method of computation explained in the preceding does not apply.

In the case of continuum structures, the direct method must be used. In other words, all reactions at the supports due to prestressing must be computed and then applied to the continuum as a loading to determine the hyperstatic actions, as done in the direct method.

The concept and procedure is described through the example of a two-way post-tensioned floor system described next.

2.2.2 Example of a Two-Way Floor Slab

Consider the case of a simple floor system shown in Fig. 2.2.2-1. The slab is resting on eight square perimeter columns and a central round column. Other features of the slab include a drop panel over the central column, and an opening. The tendon layout of the floor is displayed in Fig. 2.2.2-2. The tendons are banded along the grid lines 1, 2 and 3, and are distributed uniformly in the transverse direction. Other particulars of the model are listed below.

Post-tensioning:
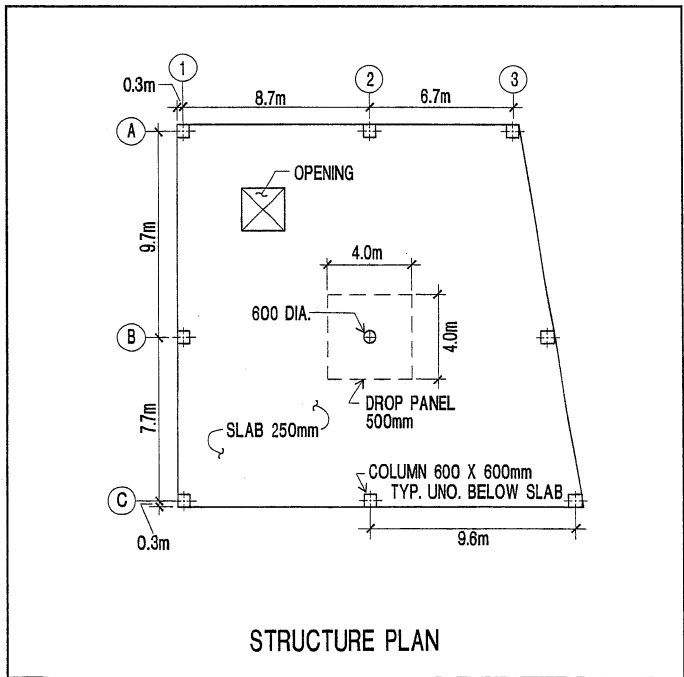
Effective Stress, f_{se} = 1400 MPa
Strand Area = 98 mm^2

STRUCTURE PLAN

FIGURE 2.2.2-1

NOTE: ● TENDON CGS ABOVE SLAB SOFFIT:
50mm AT LOW POINT
200mm AT HIGH POINT
125mm AT SLAB EDGE
▶— STRESSING END

● DISTRIBUTED TENDONS: 3 STRANDS EACH, UNO.
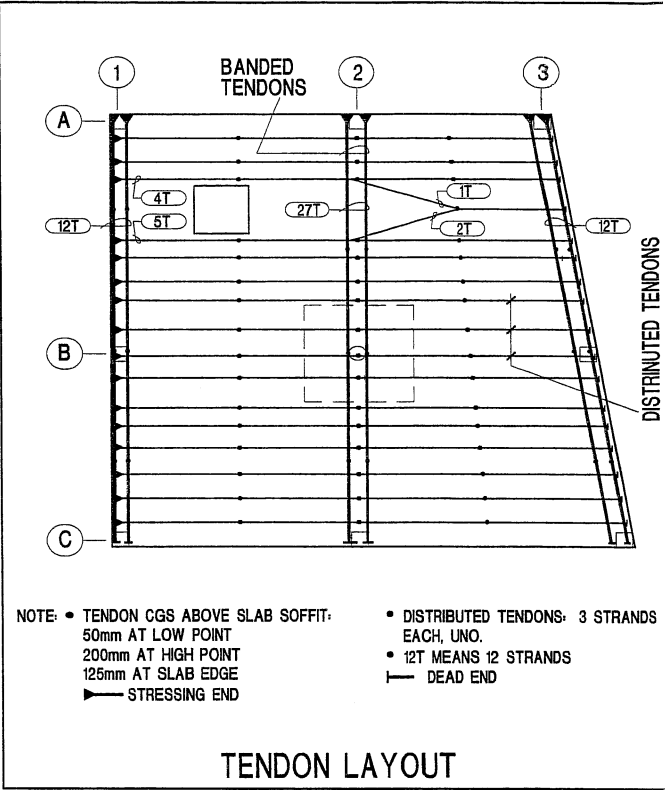● 12T MEANS 12 STRANDS
├— DEAD END

TENDON LAYOUT

FIGURE 2.2.2-2

Force assumed uniform along tendon length.
The floor was analyzed using the ADAPT-Floor software package, finite element program with the full capability to account for post tensioning. A three-dimensional, computer generated view of th structure is shown in Fig. 2.2.2-3 and the tendon layout is shown i Fig. 2.2.2-4.
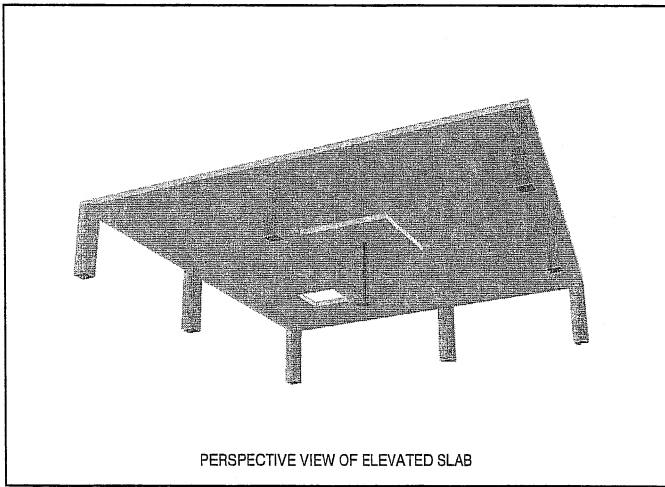

PERSPECTIVE VIEW OF ELEVATED SLAB

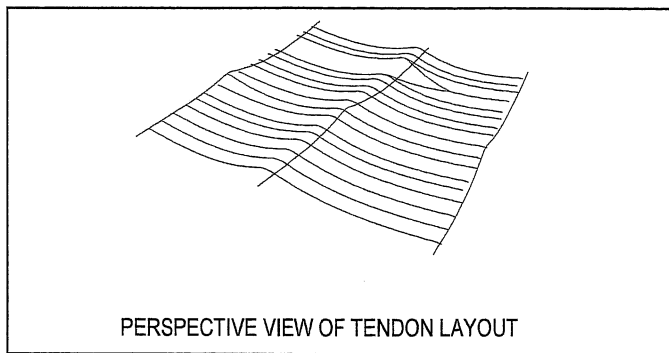**FIGURE 2.2.2-3**

PERSPECTIVE VIEW OF TENDON LAYOUT

FIGURE 2.2.2-4

For the tendon layout and associated pre-stressing, the computed hyperstatic reactions are listed in Fig. 2.2.2-5.

The algorithm used to determine the hyperstatic actions is based on the direct method described in the preceding. The principal steps are as follows:

• Calculate the post-tensioning moments and reactions in the continuum structure from the tendon layout and tendon force. The reactions at the supports due to prestressing are the hyperstatic (secondary) reactions. For the particular example, these reactions are shown in Fig. 2.2.2-5. Note that the reactions due to prestressing are in static equilibrium:

    S Secondary Actions in the vertical direction
       = 1.05 + 20.3 + 2.18 + 47.8 - 140 + 38.8 -
         0.576 + 21.6 + 8.61 kN
       = -0.236 kN ≈ 0 kN
        Vertical forces are in equilibrium.

    S Secondary Moments about Gridline B
       = (1.05 + 20.3 + 2.18) kN * 9.7 m -
        (-0.576 + 21.6 + 8.61) kN * 7.7 m
       = 0.059 kNm ≈ 0 kN-m

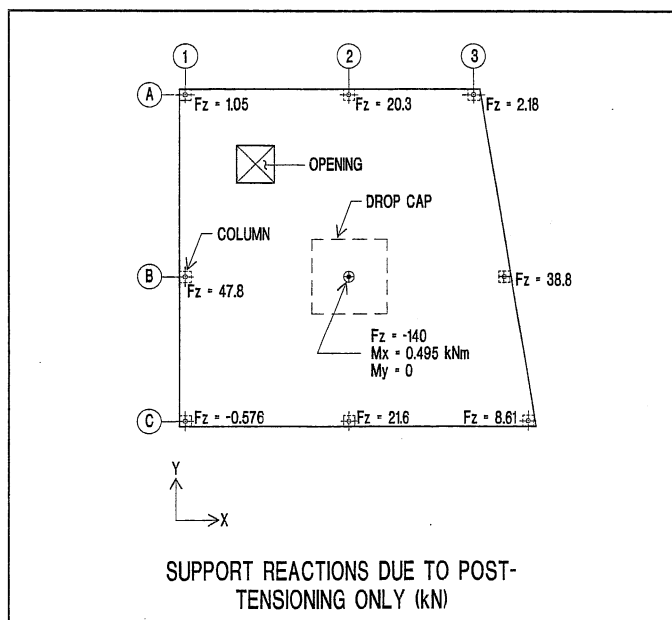
SUPPORT REACTIONS DUE TO POST-
TENSIONING ONLY (kN)

FIGURE 2.2.2-5

M_y Moments are in equilibrium.

• Use the hyperstatic reactions (shown in Fig. 2.2.2-5) as an applied loading to the continuum and determine the resulting actions within the continuum. For the floor slab under consideration, the calculated contour plot of moments, M_y, is shown in Fig. 2.2.2-6. The diagram also shows a distribution plot of the local moment per unit length about the local centroidal axis at the face of support of gridline 2. The distribution does not include moments induced by the variation of
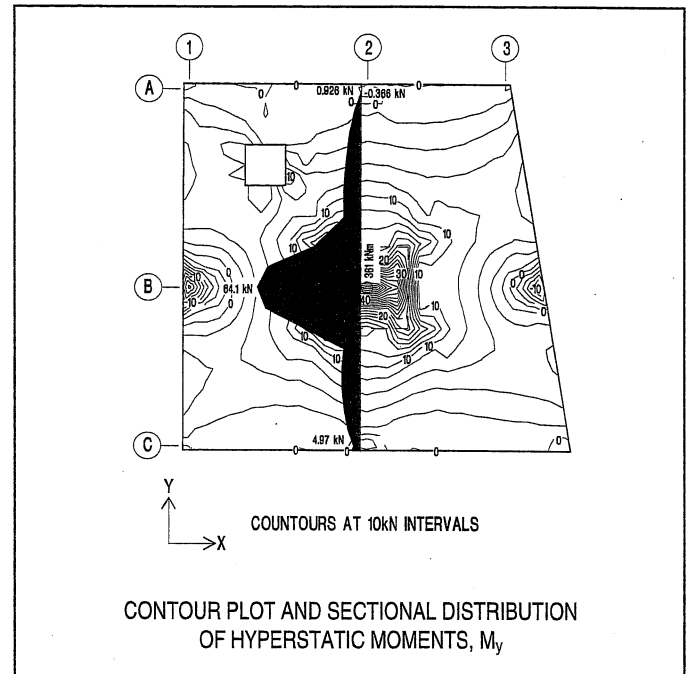

COUNTOURS AT 10kN INTERVALS

CONTOUR PLOT AND SECTIONAL DISTRIBUTION
OF HYPERSTATIC MOMENTS, M_y

FIGURE 2.2.2-6

axial forces in the slab and the shift in centroidal axis at the drop cap. It is important to note that, like other loading conditions, there will be an in-plane distribution of actions (axial and shear). These actions are shown in Fig. 2.2.2-7 for the section at grid line 2. The in-plane actions result in a tensile component in the drop panel region and compression components outside the drop panel as shown in the figure. The total hyperstatic moment at grid line 2, consists of the distribution of moment shown in Fig, 2.2.2-6 and the moment generated about the y-axis by the axial forces shown in Fig. 2.2.2-7. The sum of these two moment components is the hyperstatic moment due to pre-stressing and must balance the action due to the externally applied reactions shown in Fig. 2.2.2-5.
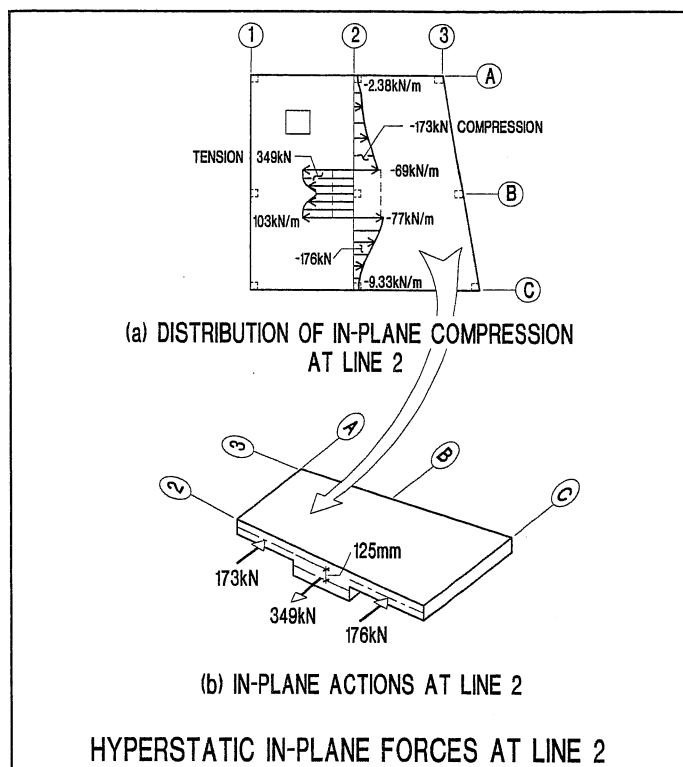
(a) DISTRIBUTION OF IN-PLANE COMPRESSION AT LINE 2

(b) IN-PLANE ACTIONS AT LINE 2

HYPERSTATIC IN-PLANE FORCES AT LINE 2

FIGURE 2.2.2-7

Fig. 2.2.2-7 shows the variation of axial forces in the model. In this case the total hyperstatic moment about the section cut is the integral of the local moment distribution (361 kN-m in Fig. 2.2.2-6) plus the integral of the local axial force distribution times the centroidal shift.

$$M_{hyp} = 361 \text{ kNm} + 349 \text{ kN} * 0.125 \text{ m} = 405 \text{ kNm}$$

This result can be verified by computing the hyperstatic moment at the face of support using the direct method.

S Secondary Moments about section drawn at Gridline 2, face of Support.
= (1.05 + 47.8 - 0.576) kN * 8.4 m
= 405 kNm

2.3 Segmental Construction

2.3.1 Introduction

A segmentally constructed structure is built over a period of time from discrete components. A distinguishing feature of the segmental construction is that, the components, or the partially assembled structure, are called upon to carry significant loading in a configuration and through a structural system other than that of the completed structure. Examples of segmental construction include balanced cantilever bridges, incrementally launched bridges, and precast prestressed girders made continuous through splicing.

In segmental construction, some or all of the prestressing is applied before the structure assumes its final configuration. Since (i) the

hyperstatic actions are caused by the resistance of the supports to the free movement of the structure, and (ii) the configuration of the structure and its support conditions change over time, the computation of hyperstatic actions can no longer be based on the geometry and prestressing of the completed structure.

In this case an incremental computational procedure must be adopted. At each stage of construction, and for each application of prestressing, the increment in hyperstatic actions must be calculated. The hyperstatic actions due to each application or change in prestressing must be accumulated to yield the total of the hyperstatic actions at any given stage.

The concepts required for computation of hyperstatic actions in segmental construction is described next through an illustrative example. For details refer to [ADAPT ABI Manual, 1997]. To focus on the hyperstatic actions, without compromising the concept, the long-term effects of creep, shrinkage, relaxation, and aging of concrete are not included in the example. A simple cross section and prestressing pattern are selected.

2.3.2 Example of a Segmental Construction

Fig. 2.3.2-1 shows the stages of construction of a two-span, continuous beam. Each span consists of a precast, prestressed girder of rectangular cross section. The girders are transported to the site and installed with a gap over the central pier (refer to the figure). A preinstalled duct in the precast girders is made continuous across the gap and the gap is cast to splice the two girders together. A post-tensioning tendon is passed through the duct. After the concrete of the splice gains adequate strength, the post-tensioning tendon is stressed, thereby achieving a continuous, two-span, post-tensioned girder. In prototype construction, a topping slab is generally applied over the precast girders, and the post-tensioning tendons are stage stressed. These operations are not addressed herein, since they do not impact the objective of the example.
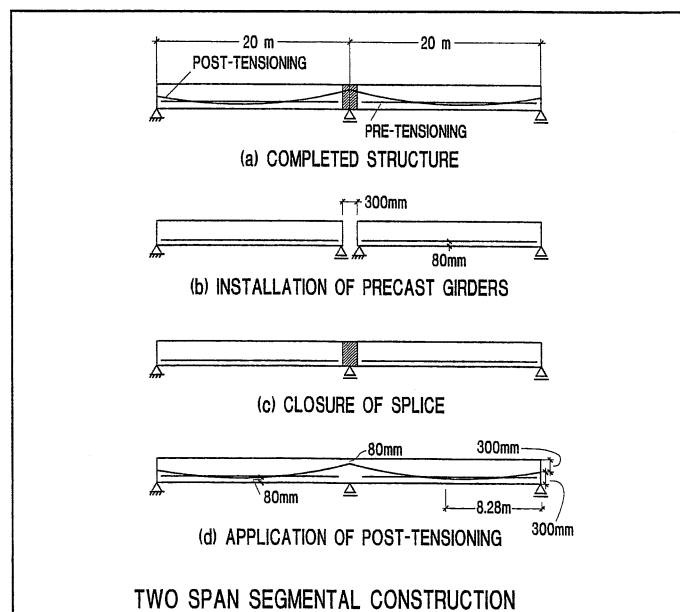

(a) COMPLETED STRUCTURE

(b) INSTALLATION OF PRECAST GIRDERS

(c) CLOSURE OF SPLICE

(d) APPLICATION OF POST-TENSIONING

TWO SPAN SEGMENTAL CONSTRUCTION

FIGURE 2.3.2-1

Details of the Structural Model:

Cross section: Rectangular 600mm deep, 250mm wide

Pre-tensioning tendons
• Force   =   1193 kN, (10 - 12mm strands)
• Location   =   CGS 80mm from the girder soffit

Post-tensioning tendons
• Force   =   1193 kN, (10-12 mm strands)
• Profile   =   Simple parabola

The distribution of moment in the completed structure due to the action of the entire amount of prestressing is shown in Fig. 2.3.2-2. The non-zero values of moment at the first and last support are due to the eccentricity of the pretensioning strands at these locations. The sharp rise in the prestressing moment over the central support is due to the fact that the pretensioning strands do not extend into the splice. At the interface of the splice, there is a drop in the moment equal to the contribution of the prestressing strands.

262 kNm

344 kNm

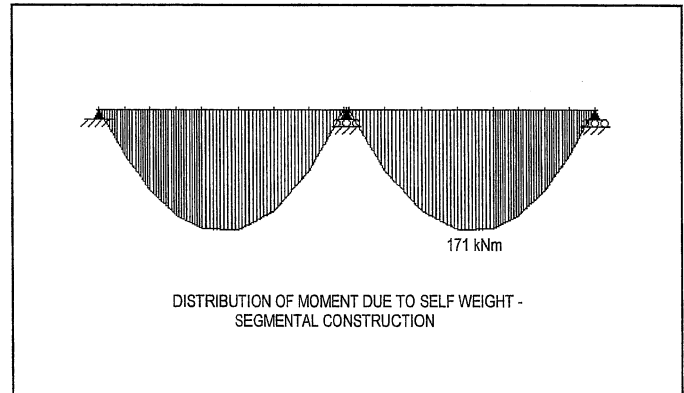DISTRIBUTION OF PRESTRESSING MOMENT IN THE COMPLETED STRUCTURE
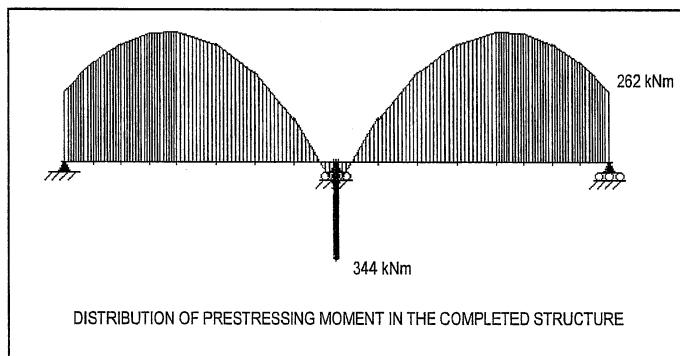
FIGURE 2.3.2-2

Fig. 2.3.2-3 illustrates the hyperstatic moments for the completed structure, based on the post-tensioning forces only. The hyperstatic (secondary) moments are due to the change in prestressing after the structure becomes indeterminate. In this example, the change is limited to the addition of the post-tensioned tendons because the pretensioning tendons were stressed when the structure was still determinate. In an actual structure, there is also a contribution to hyperstatic moments due to the stress loss in prestressing strands subsequent to the closure of the splice.

82.3 kNm

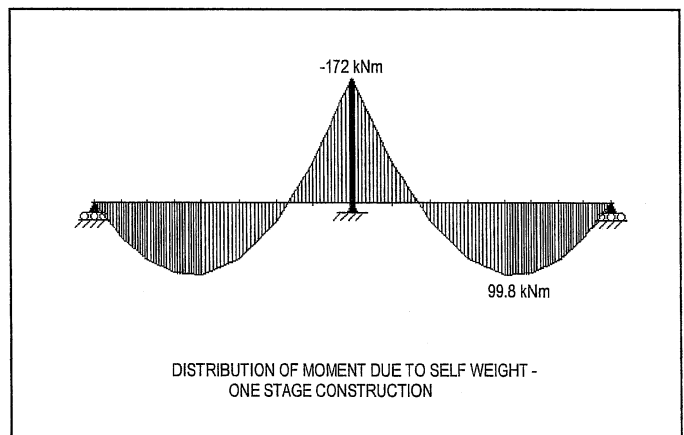DISTRIBUTION OF HYPERSTATIC MOMENTS IN THE COMPLETED STRUCTURE
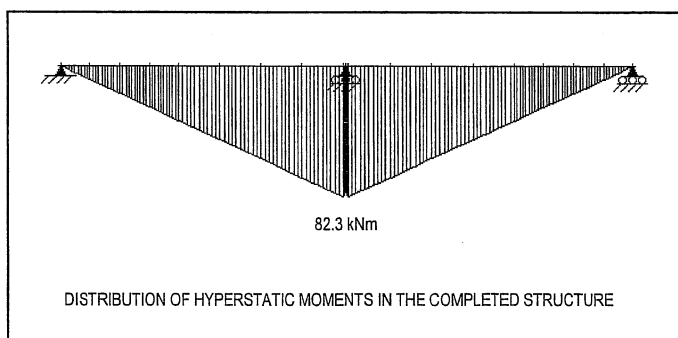
FIGURE 2.3.2-3

If the entire prestressing were considered to have been made continuous spans, the resulting hyperstatic moments would have been very different. The concept can be illustrated using the analogous example of self-weight moments in the structure for the two cases of single-stage, and segmental construction. Refer to Fig. 2.3.2-4 and 2.3.2-5. Fig. 2.3.2-4 shows the bending moment in the completed structure if it were constructed segmentally. Note that the distribution is the same, as if the two spans were simply supported. However, in a one-stage construction (shown in Fig. 2.3.2-5), the distribution of moment is due to that of a two-span continuous beam. The difference is significant.

171 kNm

DISTRIBUTION OF MOMENT DUE TO SELF WEIGHT - SEGMENTAL CONSTRUCTION

FIGURE 2.3.2-4

-172 kNm

99.8 kNm

DISTRIBUTION OF MOMENT DUE TO SELF WEIGHT - ONE STAGE CONSTRUCTION

FIGURE 2.3.2-5

The following illustrates the computation of the hyperstatic actions, using both the direct and the indirect methods. The important point to observe is that, in the computation presented, only the post-tensioning is included

A - Direct Method, Using Reactions Due to Post-Tensioning:

The reactions due to post-tensioning alone are calculated using standard procedures described earlier. The results are:

Location	Horizontal	Vertical	Moment
First support	0	$-4.1206*10^3$	0
Middle support	0	$8.2412*10^3$	0
Right support	0	$-4.1206*10^3$	0
TOTAL	**0**	**0**	**0**

From the definition of hyperstatic moments:

$$\begin{aligned}
\text{Moment over central pier} &= (\text{reaction})*(\text{span length}) \\
&= -4.1206*10^3*20*1000 \\
&= 8.24*10^7 \text{ Nmm}
\end{aligned}$$

B - Indirect Method, Using the Sectional Actions:

$$\begin{aligned}
\text{Hyperstatic moment} &= (\text{Prestressing moment}) - \\
&\quad (\text{Prestressing Force})*(\text{eccentricity}) \\
\text{Hyperstatic moment} &= (3.44*10^8 \text{ MPa}) - (1.19*10^6 \text{ N})*(220\text{mm}) \\
&= 8.24*10^7 \text{ Nmm}
\end{aligned}$$

3 - CONCLUDING REMARKS

The hyperstatic (secondary) actions in prestressed structures are defined. Two methods for their computations, referred to as Direct and Indirect Methods, are presented. It is concluded that the Direct Method is general, and that it is applicable to skeletal, continuum, and segmentally constructed structures.

It is demonstrated that, for the evaluation of the strength limit state of prestressed structures, the hyperstatic actions need to be included as part of the factored demand actions. This conclusion is contingent upon the actions having been calculated using elastic theory. Any redistribution of moments must be limited to values permitted for elastically calculated moments (typically 20%). The inclusion of hyperstatic moments in the factored demand actions does not apply, if actions are computed using plastic (yield line) theories.

An example for a floor slab, treated as a continuum structure, illustrates the development of hyperstatic in-plane forces in floor slabs, even when these are supported on rollers. This phenomenon is absent in roller supported skeletal structures.

4 - NOTATIONS AND GLOSSORY OF TERMS

e	=	Tendon eccentricity; distance from the tendon CGS to the section centroid.
M_{hyp}	=	Hyperstatic moment
P	=	Prestressing force in the plane of the slab
V_{hyp}	=	Hyperstatic shear
£	=	Strength reduction factor

Post-tensioning Moment = Primary Moments + Hyperstatic Moments
Primary Moments = Post-tensioning force * eccentricity (P*e)

TYP	=	Typical, applicable to all similar conditions
UNO	=	Unless Noted Otherwise

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